PROPER TIME EXPERIMENTS IN GRAVITATIONAL FIELDS WITH ATOMIC CLOCKS, AIRCRAFT, AND LASER LIGHT PULSES

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INTRODUCTION

Quantum Optics is a part of the more general subject of Quantum Electronics which includes atomic clocks as well as lasers. By utilizing our understanding of the quantum mechanical properties of ground state hyperfine transitions at microwave frequencies in certain atoms, very stable clocks have been made which allow highly accurate time measurements. Similar knowledge of optical transitions between electronic energy states in atoms allows lasers to be made with their many marvelous properties. These include the ability to produce very, very short pulses of light which can be used for optical radar and remote time comparison.

By combining these techniques, we have the capability of making direct measurements of distance and time of sufficient accuracy to measure the predicted effects of General Relativity in "human scale" situations on the earth.

In this talk, three different types of proper time experiments on the earth, which colleagues and I have recently carried out,[1] will be discussed:

1) local experiments in which clocks are raised to a higher gravitational potential by an aircraft and compared with clocks on the ground by short pulses of laser light;

2) global transport of clocks over a large latitude difference to study the combined effects on time of rotational velocity and gravitational potential on the rotating oblate earth;

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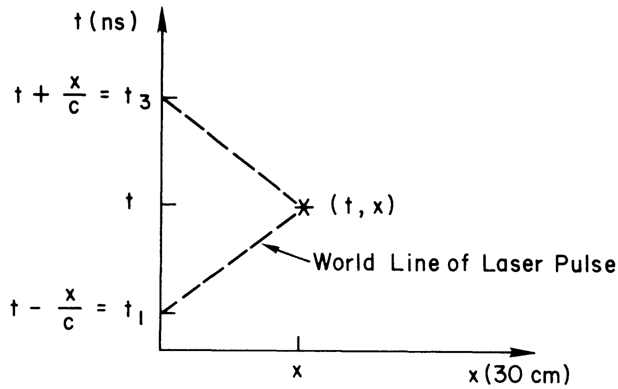

Fig. 1. Minkowski space-time diagram showing Einstein's prescription for comparing separated clocks.

and,

3) global transport of clocks between northern and southern hemispheres at the time of the summer solstice, using the earth to represent "Einstein's Elevator" falling freely in the gravitational field of the sun.

To set the experiments in context and to acquaint members of the Quantum Optics community with some of the fundamental concepts of General Relativity, these will be presented in the historical way, as I understand it, in which Einstein arrived at them.[2]

EINSTEIN'S LIGHT PULSE PRESCRIPTION FOR COMPARING SEPARATED CLOCKS

In Einstein's 1905 paper on "The Electrodynamics of Moving Bodies", he stated the ideas now known as "special" relativity -- the observers being restricted to the class of inertial observers, with gravity being ignored. Central to these ideas was his realization that time is not absolute and that the simultaneity of separated events is relative to the inertial observer. Acceptance of the physical reality of this was the key to the puzzles of light propagation and electrodynamics on which he had pondered since the age of sixteen, ten years earlier. Let us review the prescription he gave for comparing the readings of clocks separated from one another. This is most readily done with a space-time diagram as shown in Figure 1. (Einstein himself did not use such diagrams; they were first introduced by Hermann Minkowski in 1907). If one plots time in units of nanoseconds and distance in units of 30 centimeters, then the world line of a light pulse plots as a line with a slope of 45 degrees. The dashed line represents a pulse that is sent out at a time $t_1 = t - x/c$, reflected at a distance x from the origin at the time t, and received back at the time $t_3 = t + x/c$. Einsteins's prescription for determining the time t of the reflection event was based on the assumption that in an inertial frame, the speed of light is the same going out as coming back. Thus the time t of the reflection event is to be identified with the mid point in time between t_1 and t_3:

$$t = t_1 + \tfrac{1}{2}(t_3 - t_1) = t_1 + \tfrac{1}{2}t_3 - \tfrac{1}{2}t_1 = \tfrac{1}{2}(t_1 + t_3). \qquad (1)$$

With the short light pulses produced by lasers, this prescription can now be realized and affords the most accurate technique known for comparing distant clocks with each other.

The same measurements of t_1 and t_3 can also be used to calculate the distance x of the reflection event in terms of the speed of light c and the elapsed time $t_3 - t_1$:

$$x = (t_3 - t_1)\, c/2. \qquad (2)$$

This equation is the basis for laser light pulse range measurements.

In the real world, actual experiments must take account of the influence of the earth's atmosphere in reducing the speed of light. If the additional delay of the light pulse on the way out is τ_{out} and on the way in is τ_{in}, the range equation becomes

$$x = \left[t_3 - t_1 - (\tau_{out} + \tau_{in})\right] c/2 \qquad (3)$$

and one must know the delay in order to obtain an accurate measurement of distance by the radar technique.

This is to be contrasted with the short light pulse time comparison. Equation (1) becomes

$$t = \tfrac{1}{2}(t_1 + t_3 + \tau_{out} - \tau_{in}). \qquad (4)$$

The delays τ_{out} and τ_{in} are very nearly equal so that one can usually ignore atmospheric effects.

There is another correction to equation (1) which should be mentioned for observers attached to the surface of the rotating earth. These are not inertial observers since they are subject to acceleration. A consequence of this is that the coordinate speed of light in the direction East → West is different from the speed

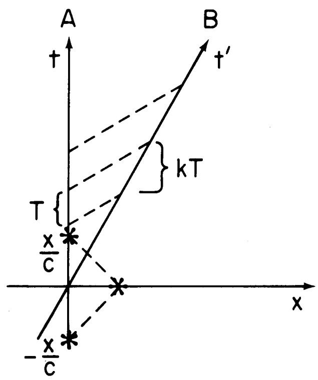

Fig. 2. Bondi's k factor (Doppler factor).

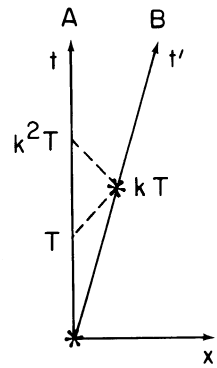

Fig. 3. Derivation of the expression for k.

of light in the direction West → East. *

BRIEF REVIEW OF SPECIAL RELATIVITY

The k-Calculus

I will rapidly review some of the notions of special relativity in a way advocated by Hermann Bondi[3] which allows a later straight forward incorporation of gravity in terms of Einstein's Principle of Equivalence.

A modern observer will be equipped with an atomic clock, a short pulse laser, a fast photo-detector, and an event timer capable of registering the epoch of arrival of a light pulse. Let us imagine two such observers, A and B, whose world lines are shown in the Minkowski diagram of Figure 2. The coordinate axes are those of A. The motion of B is away from A with constant velocity. Let A send a succession of laser pulses to B separated in time by the interval T. It is clear that there will be a longer time interval, kT, between the reception of the pulses by

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* The effective speed of light, c_{eff}, referred to the earth's surface in the E-W or W-E directions is

$$c_{eff} = c(1 \pm r\omega/c)$$

where r is the distance from the spin axis and ω is the angular velocity. At the equator $r\omega/c = 1.6 \times 10^{-6}$.

B, since he is moving and the second pulse has farther to go. Professor Bondi has developed this approach into what I regard as the most satisfactory elementary introduction to the ideas of special relativity. He calls it the "k-calculus". The factor k is, of course, just the relativistic Doppler factor. Its value is easily derived. Refer to Figure 3 where the first pulse is sent from A to B when the two observers coincide (at which time they set their identical clocks to each read zero.) The second pulse is emitted by A at time T and received by B at time kT. The pulse is reflected back to A and received by him at time $k(kT)=k^2T$. Then, by equation (2),

$$x = (k^2T - T)\ c/2 = T\ (k^2 - 1)\ c/2, \qquad (5)$$

and the time for the reception of the pulse according to A, by equation (1), is

$$t = \tfrac{1}{2}\,(T + k^2\ T) = \tfrac{1}{2}\,(k^2 + 1)\ T. \qquad (6)$$

Now the velocity v of B with respect to A is

$$v = x/t = c(k^2 - 1)/(k^2 + 1). \qquad (7)$$

Solving for k,

$$k = [\ (1 + v/c)\ /\ (1 - v/c)\ ]^{1/2}. \qquad (8)$$

How does A determine events which are simultaneous with his origin event t=0? He can send out a short pulse at time -x/c and get it returned at time x/c by reflection from the event. Then equation (1) gives the event the time coordinate 0. The locus of all such events constitutes A's spatial axis in the space-time diagram, his x-axis. This is shown in Figure 2.

If B follows the same procedure, one has the situation illustrated in Figure 4. Since the speed of light, represented by the dashed lines at 45°, is the same for B as for A, the geometry shows that B's axis of simultaneity, his x'-axis, must be tilted as shown. The angle of tilt between the x' and x axes is the same as that between the t' and t axes, namely $\tan^{-1}(v/c)$.

This is Minkowski's way of representing different inertial observers in a single space-time diagram. If an observer C moves to the left with respect to A, his time axis t runs upward to the left and his axis of simultaneity or space axis runs down to the right as shown in Figure 5. I like to think of these space and time axes as blades of a pair of scissors hinged at the common origin and "slicing up" space-time in different ways for different inertial observers. The relative simultaneity of separated events can be graphically demonstrated in terms of Minkowski diagrams.

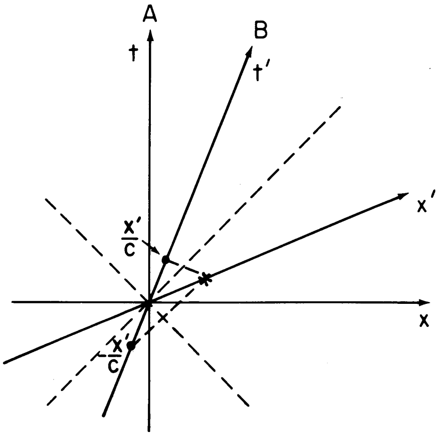


Fig. 4. Establishing an axis of simultaneity with light signals.

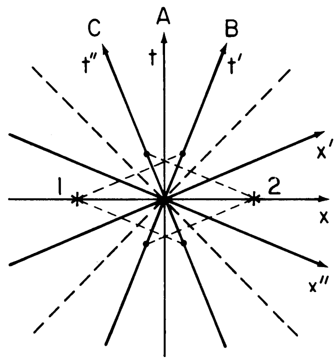


Fig. 5. Minkowski's "slicing up" of Absolute Space-Time (1907).

By projecting the time coordinates of events 1 and 2 in Figure 5 parallel to the appropriate axes of simultaneity, it is readily seen that although 1 and 2 are simultaneous for A, 2 occurs before 1 for B, and 2 occurs after 1 for C.

The most influential mathematical observation made by Minkowski in its effect on the subsequent development of relativity, was the existence of the invariant interval as it has come to be called. Imagine two neighboring events in Figure 5, separated by Δt and Δx for A, $\Delta t'$ and $\Delta x'$ for B, and $\Delta t''$ and $\Delta x''$ for C. The projection technique reveals clearly that in general,

$$\Delta t \neq \Delta t' \neq \Delta t'',$$

and

$$\Delta x \neq \Delta x' \neq \Delta x''. \tag{9}$$

However, Minkowski showed that there is a quantity, $(\Delta s)^2$, defined below, on which they all agree:

$$\begin{aligned} (\Delta s)^2 &= c^2\,(\Delta t)^2 - (\Delta x)^2 && \text{(A)} \\ &= c^2\,(\Delta t')^2 - (\Delta x')^2 && \text{(B)} \\ &= c^2\,(\Delta t'')^2 - (\Delta x'')^2. && \text{(C)} \end{aligned} \tag{10}$$

It was this result which was the basis for the introduction of gravitation as curved space-time, as we shall discuss shortly.

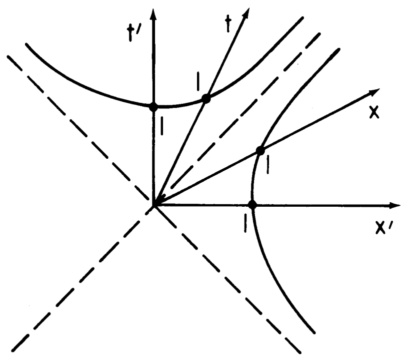


Fig. 6. Diagram for proving the invariance of the interval.

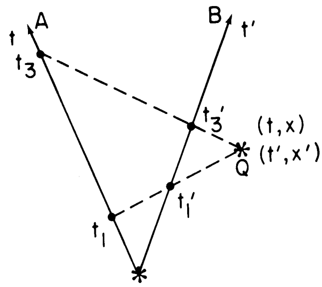


Fig. 7. Branches of the invariant hyperbolas $t'^2 - x'^2 = \pm 1$.

The invariance of the interval for all inertial observers can be simply derived using the k-calculus, in a way accessible to students in introductory physics courses. Refer to Figure 6 in which the axes of simultaneity are suppressed. One of the events is taken as the common origin event and, for simplicity, we suppress the Δ symbols. The light pulse sent by A to the reflection event Q passes by B who records its time of passage without delaying it. Then Einstein's prescription for time comparison allows us to write

$$t_1 = t - x/c, \qquad t_1' = t' - x'/c, \tag{11}$$

$$t_3 = t + x/c, \qquad t_3' = t' + x'/c.$$

The definition of the Doppler stretching factor gives

$$t_1' = kt_1', \tag{12}$$

$$t_3 = kt_3'.$$

Combining equations (11) and (12),

$$t - x/c = (1/k)\,(t' - x'/c), \tag{13}$$

$$t + x/c = k\,(t' + x'/c).$$

Multiplying these equations together, and multiplying by c^2, one obtains

$$c^2t^2 - x^2 = c^2t'^2 - x'^2 = s^2 \tag{14}$$

which is the expression for the invariance of the interval.

By solving the simultaneous equations (13) for (t, x) in terms of (t', x'), or the inverse, one obtains, of course, the Lorentz Transformation equations.

The invariance of the interval as expressed in equation (14) can be used to establish the units of length along the space and time axes for arbitrary inertial observers as shown in Figure 7. The locus of $t'^2 - x'^2 = \pm 1$ (putting c = 1) defines the hyperbola branches shown in the drawing. The intersection points for the time axes are (1, 0) and for the space axes are (0, 1). Very clear representations of time dilation and length contraction can be shown in terms of Figure 7 by projecting events onto different sets of axes.

This is the appropriate place to remind you of the famous quotation of Minkowski from a talk given in 1908:[4]

> "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics and therein lies their strength. Henceforth, space by itself and time by itself are doomed to fade away into more shadows and only a kind of union of the two will preserve independent reality."

He is, of course, talking about the invariance of the interval under the "slicing up" of space-time for different inertial observers.

It has been emphasized by Professor Eugene Wigner and others that the light pulse and timing techniques we have described can be used to measure spacelike, lightlike, or timelike intervals between events. Figure 8 shows the world line of an inertial observer which includes the event P. The event Q has a spacelike relation to P, but this is only for the convenience of showing the time intervals τ_1 and τ_2 as positive. The square of the interval between P and Q is

$$s^2 = c^2(t_P - t_Q)^2 - (x_P - x_Q)^2. \qquad (15)$$

The time intervals τ_1 and τ_2 are defined as

$$\tau_1 \equiv t_P - \left[t_Q - \frac{(x_Q - x_P)}{c}\right] = t_P - t_Q + \frac{x_Q - x_P}{c}, \qquad (16)$$

$$\tau_2 \equiv \left[t_Q + \frac{(x_Q - x_P)}{c}\right] - t_P = -(t_P - t_Q) + \frac{x_Q - x_P}{c}. \qquad (17)$$

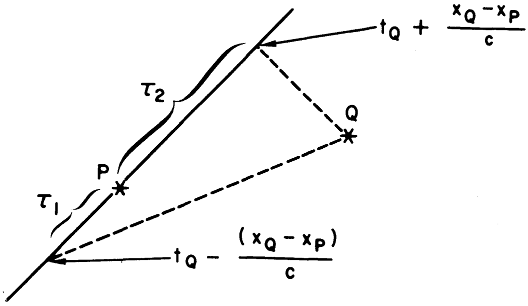


Fig. 8. The square of the interval between events P and Q: $s^2 = -c^2\tau_1\tau_2$.

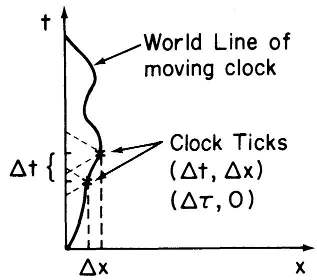


Fig. 9. The effect of motion on clocks.

By multiplying equations (16) and (17) together and multiplying by c^2, we obtain

$$c^2\tau_1\tau_2 = -s^2. \tag{18}$$

Effect of Motion on Clocks

It is customary to give a special name to the time actually kept by a moving clock: its own time, or its proper time. We will give it the symbol τ. The relation between the proper time of an arbitrarily moving clock and the time of an inertial observer who is observing the moving clock, which we will call the coordinate time, is most readily established using the invariance of the interval. In Figure 9 there are shown two clock ticks on the world line of the moving clock. The inertial observer, using his light pulse technique as shown, will measure a coordinate time interval Δt between the ticks, and a corresponding coordinate distance interval Δx between the neighboring tick events. For an observer moving with the clock, there will be an interval of proper time $\Delta\tau$ between the ticks, but there will be no spatial interval since the origin of the moving observer always coincides with the moving clock. The invariance of the interval states that

$$(\Delta s)^2 = c^2 (\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 \tag{19}$$

for two different inertial observers. We identify the primed coordinates with the inertial system moving instantaneously with the moving clock. Then, we can identify $\Delta t'$ with $\Delta\tau$, and can place $\Delta x' = 0$. Furthermore, $\Delta x = v\Delta t$ where v is the instantaneous velocity of the moving clock. We then obtain

$$(\Delta s)^2 = c^2(\Delta\tau)^2 = c^2(\Delta t)^2 - (v\Delta t)^2$$

$$(\Delta\tau)^2 = (1 - v^2/c^2)\,(\Delta t)^2 \tag{20}$$

$$\underset{\substack{\text{proper}\\\text{time}\\\text{interval}}}{\Delta\tau} = \left[1 - v^2/c^2\right]^{1/2} \underset{\substack{\text{coordinate}\\\text{time}\\\text{interval}}}{\Delta t}$$

If two similar clocks next to each other are adjusted to have the same rate ("syntonized") and set to read the same time ("synchronized"), they will not exhibit the same reading after being separated and experiencing different routes in space-time before being brought together again. Their rates, however, will again be the same. The situation is sketched in Figure 10. One can integrate equation (20) for each clock:

$$\tau_{A\ \text{final}} - \tau_{A\ \text{initial}} = \int (1 - v_A{}^2 / c^2)^{1/2} dt, \tag{21}$$

$$\tau_{B\ \text{final}} - \tau_{B\ \text{initial}} = \int (1 - v_B{}^2 / c^2)^{1/2} dt.$$

There is a "route-dependence" of elapsed proper time (in the phrase of Hermann Bondi) since the instantaneous velocities v_A and v_B will be different functions of time. To paraphrase a once popular song, "Your time is not my time"!

"Clock Hypothesis"

It is important to note the absence of any explicit dependence of elapsed proper time on the acceleration of the clock, or on any of the higher derivatives of the motion. Only the instantaneous velocity enters the equation. In the literature of relativity this is sometimes referred to as the "clock hypothesis". There is strong experimental support for this in the measurements of the decay lifetime enhancement of rapidly moving muons in a storage ring at the Center for Nuclear Research in Geneva conducted by Farley, et. al. (1966).[5] The lifetime enhancement by a factor of 12 was measured with an accuracy of ~2%. This corresponds to $v/c \approx 0.997$ and a centripetal acceleration of $v^2/r \sim 10^{15}$ m/sec^2. There is also evidence from the temperature dependence of Mössbauer nuclear γ-ray frequencies in solids. The thermal motion of the nuclei is associated with similarly large accelerations and yet equation (20) seems to be valid. These effects had to be understood and included in the Mössbauer gravitational frequency shift measurements of Pound and Rebka.[6]

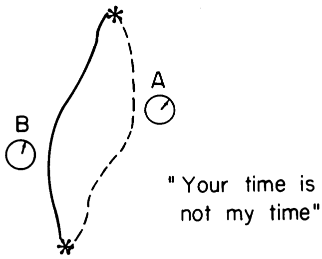


Fig. 10. The "route-dependence" of proper time.

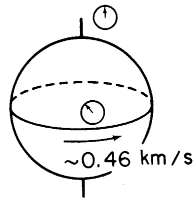


Fig. 11. Einstein's (Wrong) prediction.

For actual macroscopic atomic clocks the aircraft experiments to be described later in this talk also support the "clock hypothesis". However, it must be emphasized that macroscopic clocks can have their rates influenced by sufficiently large accelerations which, for example, can distort microwave cavities, or influence clock behavior in many other ways. These can be countered (and were) by keeping accelerations small (e.g., large turning radii), by careful packaging to isolate the clocks from vibrations and shocks, and by the careful construction of the clocks themselves.

Einstein's (Wrong) Prediction in 1905

There is the following prediction in Einstein's 1905 paper "On the Electrodynamics of Moving Bodies":

> "Thence we conclude that a balance-clock* at the equator must go more slowly, by a small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.
>
> *Not a pendulum clock, which is physically a system to which the earth belongs. This case had to be excluded."

The situation is sketched in Figure 11. The equatorial velocity is about 0.46 meters per second. Evaluating Equation (20) in an inertial (non-rotating) system with origin at the center of the earth, we find to first order,

$$\frac{\Delta\tau}{\Delta t} \approx 1 - v^2/2c^2 = 1 - 1.18 \times 10^{-12}. \tag{22}$$

In one day the equatorial clock is predicted to run slow with respect to the polar clock by 102 nanoseconds.

This prediction is wrong! The reason is that the influence of the gravitational potential difference between similar clocks is ignored. Two years were to elapse before Einstein formulated the Principle of Equivalence and realized its implications about the influence of gravity on time. The oblateness of the spinning earth causes a decrease in the gravitational potential as one gets nearer to the center of the earth in moving from the equator to the pole. It is a consequence of the basic ideas of General Relativity that this change in gravitational potential exactly compensates the change in the surface velocity as long as the clock is kept at the mean ocean surface.

We have recently performed an experiment in which atomic clocks were transported from Washington, D.C. to Thule, Greenland and back. Its results fully support the null prediction of General Relativity and will be described later.

It is interesting to speculate on what the consequences might have been for the acceptance of Einstein's ideas if clocks with the stability we have today ($\sim$2 nanoseconds/day), and the means of rapidly transporting them, had been available to test Einstein's prediction in 1905. The result would have been an unambiguous contradiction.

Let me conclude this summary of time in special relativity with a quotation from the Presidential Address at the American Association for the Advancement of Science in 1911 given by W.F. Magie, Professor of Physics at Princeton University:

> "I do not believe there is any man now living, who can assert with truth, that he can conceive of time which is a function of velocity."

This was six years after Einstein's paper, when most of the leading physicists had accepted his ideas on time. But even now there are papers written by people who question the correctness of the behavior of clocks as described above.

INCLUSION OF GRAVITY: THE PRINCIPLE OF EQUIVALENCE (EINSTEIN'S "HAPPIEST THOUGHT")

Let us now examine briefly the incorporation by Einstein of gravity into the structure of space-time, which produced curved space-time and the General Theory of Relativity, no longer restricted to inertial frames of reference. It has been my experience in teaching the subject that the best way to understand Einstein's theory of gravity is through the historical route he actually followed. The key physical idea came to him in 1907 when he was working on a summary essay concerning the special theory of relativity for the Yearbook for Radioactivity and Electronics.

Another eight years were to be required for the full mathematical formulation of curved space-time in terms of differential geometry and the tensor calculus. He later described his train of thought at that time in an essay written in 1919, "The Fundamental Idea of General Relativity in its Original Form." This has not yet been published but an excerpt was printed in the New York Times in 1972,[7] when the plans for editing and publishing all of his papers were announced. The last part of this excerpt follows:

> "I tried to modify Newton's theory of gravitation in such a way that it would fit into the theory. Attempts in this direction showed the possibility of carrying out this enterprise, but they did not satisfy me because they had to be supported by hypotheses without physical basis. At that point there came to me the happiest thought of my life [emphasis added] in the following form:
>
> Just as in the case where an electric field is produced by electromagnetic induction, the gravitational field similarly has only a relative existence. Thus, for an observer in free fall from the roof of a house there exists, during his fall, no gravitational field - at least not in his immediate vicinity. If the observer releases any objects, they will remain, relative to him, in a state of rest, or in a state of uniform motion, independent of their particular chemical and physical nature.* The observer is therefore justified in considering his state as one of "rest".
>
> The extraordinarily curious, empirical law that all bodies in the same gravitational field fall with the same acceleration immediately took on, through this consideration, a deep physical meaning. For if there is even one thing which falls differently in a gravitational field than do the others, the observer would discern by means of it that he is in a gravitational field, and that he is falling in it. But if such a thing does not exist - as experience has confirmed with great precision - the observer lacks any objective ground to consider himself as falling in a gravitational field. Rather, he has the right to consider his state as that of rest, and his surroundings (with respect to gravitation) as field-free.
>
> The fact, known from experience, that acceleration in free fall is independent of the material, is therefore a mighty argument that the

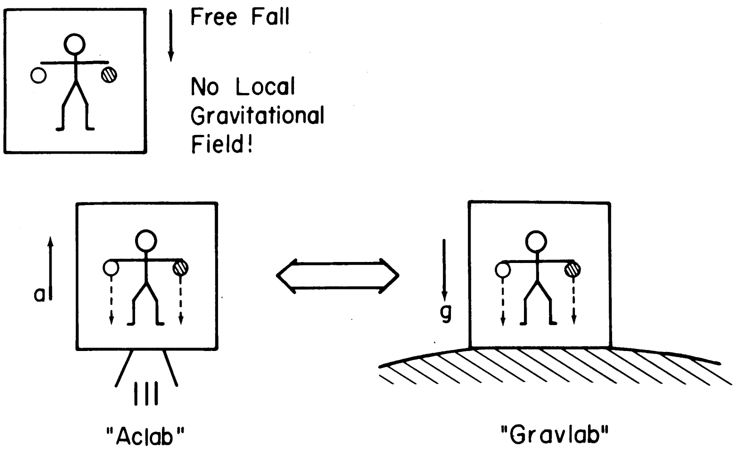


Fig. 12. Situations illustrating Einstein's Principle of Equivalence.

postulate of relativity is to be extended to coordinate systems that are moving non-uniformly relative to one another."

*In this consideration one must naturally neglect air resistance."

The dependence of proper time on gravitational potential can be deduced with very little mathematics from the Principle of Equivalence and the technique of the k-calculus. Consider a freely falling laboratory as shown in Figure 12. Objects of whatever composition will remain at rest if released with no initial velocity. The path of an object will be a straight line if there is some initial velocity. This is all very familiar to us now from the televised activities of the U.S. astronauts on the Apollo, Skylab, and Space Shuttle spacecraft and of Soviet cosmonauts in the Salyut and Soyuz spacecraft. A freely falling spacecraft constitutes a true (local) inertial system. The acceleration of gravity is only cancelled locally because of gradients in any real gravitational field.

If one imagines the spacecraft in a region of space free of gravity (strictly nowhere achievable) but subject to the acceleration produced by a rocket engine as in Figure 12, (the "Aclab" in the terminology of Banesh Hoffmann[2]), the observer inside will experience apparent gravitational effects, as objects of whatever composition appear to him to move to the floor with the same accelerated motion. This behavior would be indistinguishable (locally) from that in the "Gravlab" on the

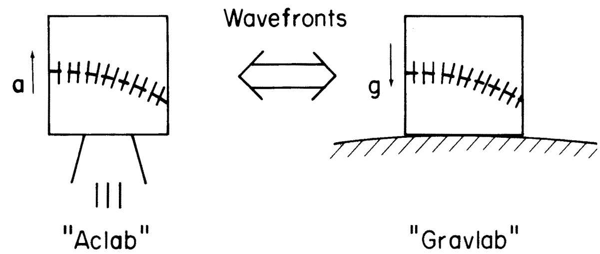


Fig. 13. Implication of the Principle of Equivalence for the propagation of light in a gravitational field.

surface of the earth or other massive body if a = g. The fractional difference between the gravitational accelerations of small amounts of aluminum and gold has been measured to be $< 10^{-11}$ in experiments at Princeton Uniersity by Dicke, et. al. (1964).[8] Using a similar technique, Braginsky, et. al. (1974)[9] at Moscow State University have measured the fractional difference for aluminum and platinum and found it to be $< 10^{-12}$. The results of the lunar laser ranging experiment[10] show that the fractional difference between the accelerations of the massive bodies earth and moon, falling toward the sun, is also $<10^{-11}$.

Speed of Light in a Gravitational Field

One can draw an immediate conclusion from the Principle of Equivalence without any mathematics. If the Principle is to hold for all of physics, not just for dynamics, then it will apply to the propagation of a light wave as shown in Figure 13. If the "Aclab" is accelerating with respect to an inertial system in which light will propagate in a straight line, then in the "Aclab" the path of the light will be curved. If the equivalence with "Gravlab" is correct, there is the immediate prediction of the curvature of a light path in a gravitational field. One can go further by using the analogy of the phase fronts of the light to

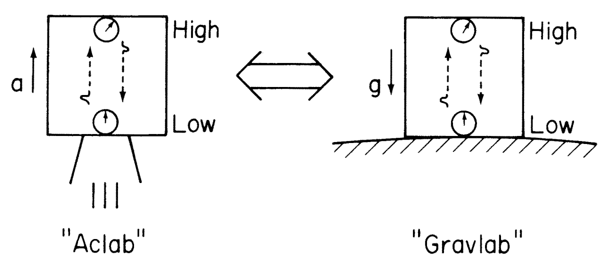


Fig. 14. Implication of the Principle of Equivalence for the behavior of clocks in a gravitational field.

the ranks of marching soldiers. To make a turn while maintaining the ranks, the outer soldiers must march faster than the inner soldiers. The conclusion is that the speed of light must increase with height in a gravitational field.

The Effect of a Gravitational Field on Clocks

In Figure 14, clocks of identical construction are shown at the floor and ceiling of "Aclab" and "Gravlab". The High and Low clocks in each instance are being compared by modern observers equipped with short pulse lasers, fast photo-detectors, and event timers as we have discussed earlier. In the next paragraph, we will use the approach of the k-calculus to calculate the rate of the High clock with respect to the Low clock for the "Aclab". The conclusion will be that the High clock will run fast with respect to the Low clock. Therefore, if the Principle of Equivalence is valid for all of physics, the High clock in "Gravlab" will run fast with respect to the Low clock. The rate of a clock is predicted to depend on its position in a gravitational field.

Figure 15 shows the curved world lines of the accelerated High and Low clocks in "Aclab" with respect to an inertial observer. The clocks are separated by a vertical distance h when t = 0. Two successive light pulses are emitted by the Low observer separated by a time interval T as measured on his clock. They will be received by the High observer with an interval kT, where the velocity v to be used in equation (8) is given approximately by v = a(h/c) where a is the acceleration of "Aclab". Then,

$$k = \left[\frac{1 + v/c}{1 - v/c} \right]^{1/2} \approx \left[1 + 2v/c \right]^{1/2} \approx \left[1 + 2ah/c^2 \right]^{1/2}. \qquad (23)$$

By the Principle of Equivalence, a = g, so,

$$k = \left[1 + 2gh/c^2 \right]^{1/2}. \qquad (24)$$

For small h, gh is just the Newtonian gravitational potential ϕ. We then have

$$k = \left[1 + 2\,\phi/c^2 \right]^{1/2}. \qquad (25)$$

The space-time diagram in "Gravlab" is shown in Figure 16. The High and Low clock world lines are straight and vertical since the clocks are at rest. The dashed light propagation lines are curved because the speed of light increases with height. The time interval T becomes stretched to kT as a function of vertical distance x in the local gravitational field,

$$k = \left[1 + 2gx/c^2 \right]^{1/2}. \qquad (26)$$

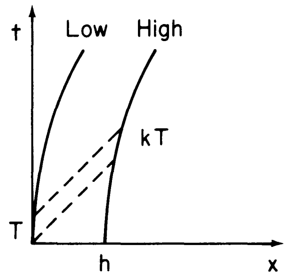


Fig. 15. Comparison of clocks in "Aclab."

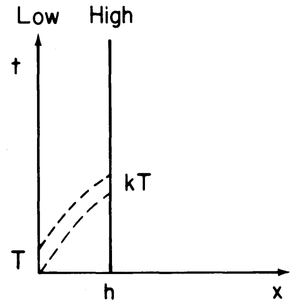


Fig. 16. Comparison of clocks in "Gravlab."

This is the first appearance of what we can call the "curvature of time" in a gravitational field. The proper time of a clock depends on where it is in the gravitational field. It is convenient again to refer to the coordinate time established by an observer at the surface of the earth, for example, by his use of light pulses. Figure 17 illustrates the procedure for the coordinate observer on the ground at $x = 0$. He follows the same "Einstein prescription" we discussed earlier to establish the coordinate time of a distant event, sending out a light pulse and getting the reflected pulse back at a later time. The mid-point between his emission and reception events is to be taken as simultaneous with the coordinate time of the reflection event. (Even though the speed of light is not constant in a gravitational field, this procedure is valid since the time required to go out is the same as the time to return.) If the procedure is repeated a coordinate time interval Δt later, as shown, the successive events at the High position will be recorded by a clock there with a proper time interval $\Delta\tau$. The important predicted physical property is

$$\Delta\tau \neq \Delta t,$$

but rather

$$\Delta\tau = (1 + 2\phi/c^2)^{1/2} \Delta t. \tag{27}$$

Time Curvature: Modification of the Invariant Interval

Einstein generalized the expression for the invariant interval in restricted relativity, equation (19), in the presence of a gravitational field, by adding a coefficient in front of $c^2(\Delta t)^2$:

$$(\Delta s)^2 = (1 + 2gx/c^2)\, c^2\, (\Delta t)^2 - (\Delta x)^2. \tag{28}$$

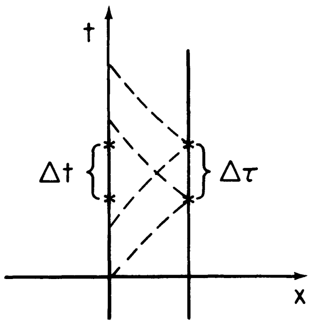


Fig. 17. Einstein's prescription for the comparison of clocks in a gravitational field.

For non-uniform gravitational fields, gx is to be replaced by the Newtonian gravitational potential ϕ:

$$(\Delta s)^2 = (1 + 2\phi/c^2)\, c^2\, (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2, \quad (29)$$

where we have now included all three spatial dimensions.

One speaks of equation (29) as the metric of space-time. This is because Einstein retained the interpretation,

$$(\Delta s)^2 = c^2\, (\Delta \tau)^2, \quad (30)$$

Where $\Delta\tau$ is the proper time interval recorded by a clock which is present at the two neighboring events separated by the interval (timelike separation). The interval is a measure of the proper time interval. A similar interpretation is to be retained for spacelike separations where Δs becomes a measure of the proper distance interval $\Delta \ell$:

$$(\Delta s)^2 = -(\Delta \ell)^2. \quad (31)$$

The factor $(1 + 2\phi/c^2)$ is called a metric coefficient.

It is clear that for static clocks in a gravitational field as in Figure 17, this form of the metric will reproduce equation (27):

$$\Delta\tau = (1 + 2\phi/c^2)\, \Delta t. \quad (32)$$

For a moving clock in a gravitational field Einstein's idea was to retain the same interpretation of $(\Delta s)^2$ which we used in deducing equation (20): it is to be the same for all observers. For the moving clock, $\Delta s = c\Delta\tau$. For the coordinate observer,

$$\Delta x = v_x \Delta t,\ \Delta y = v_y \Delta t,\ \Delta z = v_z \Delta t, \tag{33}$$

$$v^2 = v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2} .$$

Then,

$$(\Delta s)^2 = c^2(\Delta\tau)^2 = (\ 1 + 2\phi/c^2)\ c^2\ (\Delta t)^2 - v^2(\Delta t)^2, \tag{34}$$

which leads to

$$\underset{\substack{\text{proper}\\\text{time}\\\text{interval}}}{\Delta\tau} = \left[\ 1 + 2\phi/c^2 - v^2/c^2\ \right]^{1/2} \underset{\substack{\text{coordinate}\\\text{time}\\\text{interval}}}{\Delta t} \tag{35}$$

as the relation between proper time interval and coordinate time interval for a clock in motion in a gravitational field.

It is the predicted relation (35) which we have been able to verify to about 1.5% in the aircraft experiments to be described shortly. For these experiments $\phi/c^2 \approx 10^{-12}$ and $v^2/2c^2 \approx 10^{-13}$ so the first order expansion of equation (35) is adequate:

$$\Delta\tau = \left[\ 1 + \phi/c^2 - v^2/2c^2\ \right]\ \Delta t. \tag{36}$$

It was also Einstein's idea to obtain the coordinate speed of light by setting

$$(\Delta s)^2 = 0, \tag{37}$$

just as in special relativity. Using equation (34), we obtain

$$c_{coord} = (\ 1 + 2\ \phi/c^2\ )^{1/2}\ c \tag{38}$$

which shows how the coordinate speed of light is to depend on the gravitational potential, giving quantitative form to our earlier qualitative considerations.*

* This expression for the coordinate speed of light had to be corrected when the complete theory of General Relativity including curved space as well as curved time was developed. Equation (38) was used by Einstein when he first predicted the gravitational deflection of starlight by the sun to be 0.84 seconds of arc for grazing incidence. The inclusion of curved space metric coefficients gives an additional factor of 2, which is verified by observations.

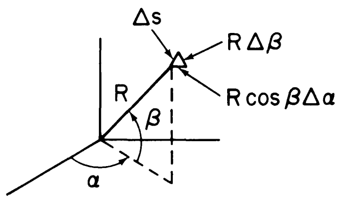


Fig. 18. Coordinate system for latitude and longitude

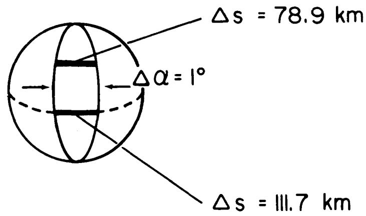


Fig. 19. Different proper distances on the sphere.

Analogy of Time Curvature to the Curvature of a Sphere

The existence of metric coefficients which multiply the square of coordinate increments to give the true measure of length, or proper length, between neighboring points on the surface of a sphere like the earth is very familiar. Figure 18 shows the coordinate system of longitude α and latitude β. The distance Δs at the radius R is given by

$$(\Delta s)^2 = R^2 \cos^2 \beta \, (\Delta\alpha)^2 + R^2(\Delta\beta)^2. \tag{39}$$

The actual increment of proper length will be different at different locations on the sphere even though the coordinate increment is the same. For example, let $\Delta\alpha$ = 1 degree, and $\Delta\beta = 0$, as shown in exaggerated fashion in Figure 19. Then,

$$\Delta s = R \cos \beta \, \Delta\alpha \tag{40}$$

becomes 112 km for $\beta = 0$ and 79 km for $\beta = 45°$. The analogy is very thorough to the relation between the increments of proper time and of coordinate time expressed in equation (32).

It is not possible here to review the remarkable developments in the study of the differential geometry of general two-dimensional curved surfaces made by Gauss. However, one discovery must be mentioned, which Gauss called a "Theorema Egregium" ("a most excellent theorem"). For the quadratic form expressing the proper length on the surface in terms of the coordinate differentials Δx_1 and Δx_2,

$$(\Delta s)^2 = g_{11}(\Delta x_1)^2 + 2\, g_{12}\, (\Delta x_1)\, (\Delta x_2) + g_{22}(\Delta x_2)^2, \tag{41}$$

The Gaussian curvature of the surface can be calculated from the way the metric coefficients $g_{ij}(x_1,x_2)$ change their values as a function of position within the surface itself. Although the Gaussian curvature involves the space outside the surface (for the sphere it is just $1/R^2$), it can be expressed in terms of derivatives of the $g_{ij}(x_1,x_2)$ on the surface. The Gaussian curvature is thus an intrinsic property of the surface which is invariant under deformations which do not involve stretching, shrinking or tearing -- a so-called bending invariant.

The extension of these ideas about curvature to n-dimensions by Riemann and their elaboration by other differential geometers such as Ricci, Christoffel, and Levi-Civita, including the tensor calculus, have played a large role in the development of General Relativity. However, the major physical idea came first -- the Principle of Equivalence.*

It is difficult to visualize the time curvature, although I hope that the aircraft experiments with atomic clocks which are the subject of this paper will help to develop intuition about it. One way to dramatize it is in terms of Figures 20 and 21. The quantity ϕ/c^2 which enters the metric coefficient for time is plotted as a function of distance from the Sun, leading to the familiar gravitational potential "well", or "hill", depending on the observer's location. It can be written

$$\frac{\phi}{c^2} = \frac{-GM_\odot}{rc^2} = \frac{-GM_\odot}{R_\odot c^2}\left(\frac{R_\odot}{r}\right) \tag{42}$$

where G = Newtonian Gravitational Constant
$M_\odot$ = Mass of the Sun
$R_\odot$ = Radius of the Sun

---

* Einstein was taught differential geometry and guided through its literature by Marcel Grossmann during the period 1912-14 when Einstein was a professor at the Eidgenössiche Technische Hochschule (ETH) in Zürich. Grossmann, a long-time friend, was a professor of mathematics at the ETH. All physicists should be aware of two other acts of friendship which Grossman did for Einstein. When they were both students at the ETH, Einstein habitually cut classes to study independently. Only through his access to Grossmann's carefully taken notes was he able to prepare for his examinations. The second act was Grossmann's intercession with his own father, who was a friend of the director of the Swiss Patent Office, to obtain for Einstein an interview at the Patent Office, which led to his seven enormously fruitful years, from 1902 to 1909, as a patent examiner. His livelihood did not depend on producing physics, so he was free to think about the most important problems of physics, without external pressure for results. See the biography by Hoffmann[2] for details.

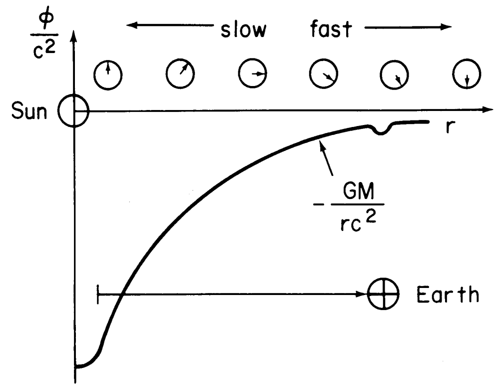


Fig. 20. The rate of a clock as influenced by the gravitational potential of the sun: "time curvature."

The depth of the solar potential well, $GM_\odot/R_\odot c^2 \approx 2 \times 10^{-6}$. A similar expression holds for the depth of the potential well of the earth, which is shown superimposed (not to scale) on the solar potential. Its value is $\approx 7 \times 10^{-10}$. With respect to some coordinate observer, a clock runs slower as it is lower in the potential well, or faster as it is higher on the potential hill.

Consider Figure 21 drawn by Herblock in 1955 after the death of Einstein. Let us imagine (granting considerable astronomical license to Herblock) that this is a view of the earth by an advanced observer who is located far out along the potential hill of the Sun. If he is equipped with good atomic clocks, then the interval of exactly 100 years between noon on the birth date of Einstein and noon at the celebration of his centennial on March 14, 1979, as measured on earth, would be 100 years plus 46 seconds for the remote observer: 29 seconds from the ascent up the potential hill of the Sun; 2 seconds from the ascent up the potential hill of the earth; and 15 seconds from the lack of participation by the observer in the orbital motion of the earth. (We assume he follows along with the proper motion of the Sun.) These numbers follow from equation (36).

BRIEF REVIEW OF THE CURVED SPACE-TIME THEORY OF GRAVITY

As Einstein developed the concepts that followed from the Principle of Equivalence and the incorporation of gravity into the metric structure of space-time during the years 1907-1915, especially the years 1912-1914 of close collaboration with Marcel Grossmann, he realized that there should be metric coefficients

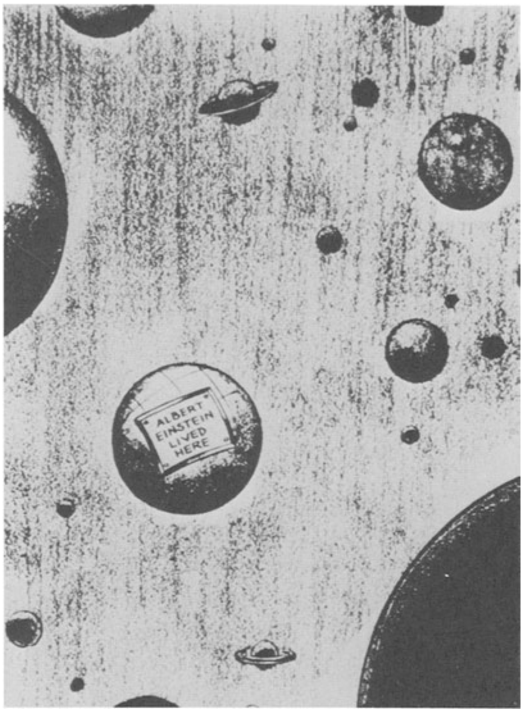


Fig. 21. Drawing by Herblock in the Washington Post after Einstein's death in 1955.

depending on gravity for the spatial terms of the metric. In other words, gravity should produce a space curvature as well as a time curvature. However, we shall see shortly that the dominant curvature for ordinary motions under gravity is the time curvature, since it alone is needed to deduce Newton's Theory from Einstein's Theory in the limiting case of weak gravity and slow motions.

Another central idea during the gestation of General Relativity was the desire to remove the restriction to inertial observers, allowing observers with any motion whatever. The invariance of the interval (now using differentials and the Einstein convention for summation on repeated indices)

$$(ds)^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu, \quad \mu,\nu = 0, 1, 2, 3 \tag{43}$$

was to be maintained for all observers. Here $dx^0 = cdt$ and the dx^i, $(i = 1, 2, 3)$, are the three spatial coordinates. Quite arbitrary coordinate transformations among the dx^μ could be admitted, with appropriate changes in the $g_{\mu\nu}$, so long as the square of the invariant interval $(ds)^2$, remained unchanged. This became known as the Principle of General Covariance. For neighboring events on the world line of a moving clock, $ds = c d\tau$, as before.

Motion of a particle under the influence of gravity is to be described by a "geodesic" in the curved space-time produced by gravity. That is, $\int ds$ is to be an extremum when the integral is taken between the events at the beginning and end of the motion. In this case, the extremum turns out to be a maximum since it is proportional to the elapsed proper time of a clock imagined to be attached to the moving particle. It is clear that this should be the case since the moving clock can be regarded as traversing a succession of freely falling frames of reference, each of which is a local inertial system. In such systems the motion will be in a straight line with constant velocity. There is therefore another local inertial system for which the clock is at rest. From the ideas of the route dependence of proper time discussed earlier, the elapsed time of the clock at rest will be greater than for a clock with a curved path in the local flat Minkowski space-time. Bertrand Russell has called this geodesic requirement the "Principle of Cosmic Laziness."

Newtonian Physics from Time Curvature Only

We can demonstrate at this point that the Newtonian equations for motion is a weak gravitational field with $|v/c| \ll 1$ and $|\phi/c^2| \ll 1$ follow from the geodesic principle with only the metric coefficient for time included. The geodesic condition is

$$\delta \int ds = \delta \int c\,d\tau = 0. \tag{44}$$

From equation (36),

$$\begin{aligned} d\tau &= \left[1 + \phi/c^2 - v^2/2c^2\right] d\tau \\ &= \left[1 + (1/c)^2(\phi - v^2/2)\right] dt. \end{aligned} \tag{45}$$

Now the LaGrangian per unit mass, L, of a particle is given by

$$L = v^2/2 - \phi. \tag{46}$$

Substituting in equation (44) and dropping constant terms which do not affect the variation,

$$\delta \int (-L)\, dt = 0. \tag{47}$$

It is well known that the Euler-LaGrange equations resulting from this variation are:

$$\dot{\vec{v}} = - \vec{\nabla}\phi \tag{48}$$

which are Newton's equations of motion.

This is the basis of our earlier assertion that the primary curvature of Einstein's theory should be regarded as that of time. Of course, the inclusion of all the metric coefficients is needed for motion in strong fields or with large velocities, and for the calculation of departures from Newtonian physics as in the case of the 43 seconds of arc per century precession of the perihelion of Mercury.

It is very interesting to note that in the nineteenth century the mathematicians B. Riemann and W. K. Clifford, and also, I believe, the physicist L. Boltzmann, all attempted to construct a theory of gravity based on space curvature. They were not successful because they missed completely the concept of time curvature.

The Field Equations

The final problem to be solved by Einstein in arriving at the full description of General Relativity was the choice of appropriate field equations for the metric coefficients $g_{\mu\nu}$ in the neighborhood of masses. After several false starts, he finally chose a set of 10 second order partial differential equations for the 10 (symmetric) $g_{\mu\nu}$. The equations are non-linear in that they involve products of first derivatives of the $g_{\mu\nu}$. We display them in their familiar symbolic form

$$R_{\mu\nu} - \tfrac{1}{2} Rg_{\mu\nu} = (8\pi G/c^4)\ T_{\mu\nu}. \tag{49}$$

The source term on the right, $T_{\mu\nu}$, is the general stress-energy tensor which includes mass, energy, stresses, etc., all of which produce gravitational fields in Einstein's Theory. On the left-hand side are two curvatures from differential geometry involving first and second derivatives of the $g_{\mu\nu}$ with respect to time and space. $R_{\mu\nu}$ is the contracted Riemann Curvature Tensor, called the Ricci Tensor, and R is the Curvature Scalar obtained from the Ricci Tensor by contraction. Unfortunately, there is no time to describe them in any more detail. They can be contrasted with the more elegant and concise form given in terms of modern differential geometry by Professor Wheeler in his lectures at this Advanced Study Institute.

Soon after Einstein proposed the full equations, an exact solution was obtained by the astronomer Karl Schwarzschild for the $g_{\mu\nu}$ outside a non-rotating spherical mass. Let us display it:

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)c^2(dt)^2 - \frac{(dr)^2}{\left(1 - \frac{2GM}{rc^2}\right)} - r^2\cos^2\beta(d\alpha)^2 - r^2(d\beta)^2 . \tag{50}$$

The major solar system observational and experimental tests of General Relativity -- the deflection by the sun of starlight and of radio emission from quasars (and the related direct measurement of extra time delays in radar ranging to the planets), the perihelion precession of Mercury and other planets, the lunar laser ranging measurements, and the solar red shift measurements -- are analyzed using this basic form of the metric. The "parametrized, post-Newtonian" form of the metric currently fashionable in analyzing tests can be regarded as based on the Schwarzschild solution.

EXPERIMENTAL MEASUREMENTS OF GENERAL RELATIVISTIC EFFECTS ON TIME

For completeness, before describing our experiments and measurements with aircraft and atomic clocks, it is appropriate to give you references to some other related experiments. A more comprehensive survey of other experiments, including also special relativity, is contained in the author's survey paper, Relativity and Clocks.[1]

The first terrestrial measurement of the gravitational red shift was made by R. V. Pound and G. A. Rebka (1960)[6] using the newly discovered technique by Mössbauer, of high resolution γ-ray spectroscopy in solids. The initial measurements of the frequency shift were accurate to about 10%. They were later extended to 1% accuracy by Pound and J. Snider (1965).[11]

Three measurements of the Solar redshift have been made in recent years by direct comparison of spectral lines with laboratory sources. The accuracy is about 5% in each case. The experiments were done at Princeton University by J. Brault (1963)[12], in France by J. Blamont and F. Roddier (1965)[13], and at Oberlin College by J. Snider (1972)[14].

In 1971, J. C. Hafele and R. E. Keating[15] demonstrated the relativistic effects on time for macroscopic cesium beam atomic clocks by carrying an ensemble of four of them on commercial aircraft flights around the world, first eastward and then westward. This allowed the large asymmetric contribution of the earth rotation velocity to be exploited. There were many uncertainties associated with the knowledge of the velocity, position, and altitude of the various aircraft from which to calculate the relativistic proper time integral. There were also problems with the lack of environmental control for the clocks. Changes in rate were identified for some clocks with respect to the average by intercomparing all clocks once per hour during the flights. The rate changes were corrected for with a systematic error given as ±30 ns. The following predicted effects were published:

EFFECT	EASTWARD	WESTWARD
Potential	144 ± 14 ns	179 ± 18 ns
Velocity	-184 ± 18 ns	96 ± 10 ns
Net	- 40 ± 23 ns	275 ± 21 ns

For measured values, they give:

EASTWARD	WESTWARD
-59 ns	+273 ns.

These values should presumably have associated with them at least the ±30 ns quoted systematic errors in the rate correction procedure. There is no doubt that the existence of the effects was demonstrated.

Mountain to valley comparisons using clock transport have been made in 1975 in Italy by L. Briatore and S. Leschiutta[16] with a height difference of 3250 m and a dwell time of 66 days. A measurement accuracy of 15% was achieved. Similar measurements were made in Japan in 1977 by S. Iijima and K. Fujiwara[17] with a height difference of 2818 m and a dwell time of one week. Systematic corrections for environmental changes and careful environmental control during clock transport allowed an accuracy of 5%.

The most accurate measurement has been done by R. F. C. Vessot and M. W. Levine (1979)[18] with a hydrogen maser in a NASA Scout rocket which rose to about 10,000 km above the earth's surface and fell back (unrecovered) into the Atlantic Ocean. The ratio of the measured to the predicted value was $1 + (2.5 \pm 70) \times 10^{-6}$, a confirmation to better than 0.01%. They measured frequency, rather than time, using a clever three-frequency scheme to cancel the very large Doppler and ionospheric efforts, which were $\Delta\nu/\nu \approx 2 \times 10^{-5}$, sufficiently well to measure to 0.01%, the gravitational potential effect of 4×10^{-10}.

LOCAL FLIGHTS WITH LASER LIGHT PULSE TIME COMPARISON

The spirit in which our experiments were done in expressed well in this quotation from a letter of Einstein to the physicist

Felix Ehrenhaft in 1939:

> "The goal of the experimental physicist is not only to obtain reproducible experimental results. The determining factors should also be as simple as possible so that one can derive from them elementary laws that one can apply to other situations."

My colleagues and I have done a series of experiments, made possible by the remarkable stability of modern atomic clocks, which we hope are a convincing demonstration of the correctness of the revolutionary views about time given to us by Einstein.

My chief collaborators have been Leonard Cutler, director of physics research at the Hewlett-Packard Company, who was in charge of the engineering design and development of the cesium atomic beam clocks which performed best among the several types of clocks used in the measurements; Robert Reisse and Ralph Williams whose Ph.D. thesis research[19,20] was an important part of the experiments; and Gernot Winkler, director of the Time Services Division of the U. S. Naval Observatory, who provided the atomic clocks and contributed much other support. There was also much help from engineers and technicians at the University of Maryland, Hewlett-Packard, and the Naval Observatory. We were also lent equipment by the Goddard Space Flight Center of the National Aeronautics and Space Administration.

Generous financial support was provided by the U. S. Navy and U. S. Air Force because of the practical need to understand and to apply relativistic corrections in the engineering of modern navigational and timekeeping systems. One of these applications is discussed in the last section of the paper.

Figure 22 is a schematic diagram of the local experiments conducted at the Patuxent Naval Air test Center with a Navy P3C anti-submarine patrol plane (military version of the Lockhead Electra) which is shown in flight in Figure 23. It is capable of staying aloft for 15 or 16 hours continuously at altitudes up to 10,700 meters.

On board were three Hewlett-Packard cesium atomic beam clocks and three Efratom rubidium optical pumped gas cell atomic clocks, carefully packaged in a "clock box" to shield them from environmental changes. The plane also carried a fast photodetector and an event timer so that the epoch of the arrival of short pulses of laser light could be registered in the time scale of the airborne clocks. This information was stored in a minicomputer system. There was an optical corner reflector (identical to those placed on the moon for the lunar laser ranging measurements) on either side of the aircraft to reflect back the

Fig. 23. The Navy P3C aircraft used for the local flights.

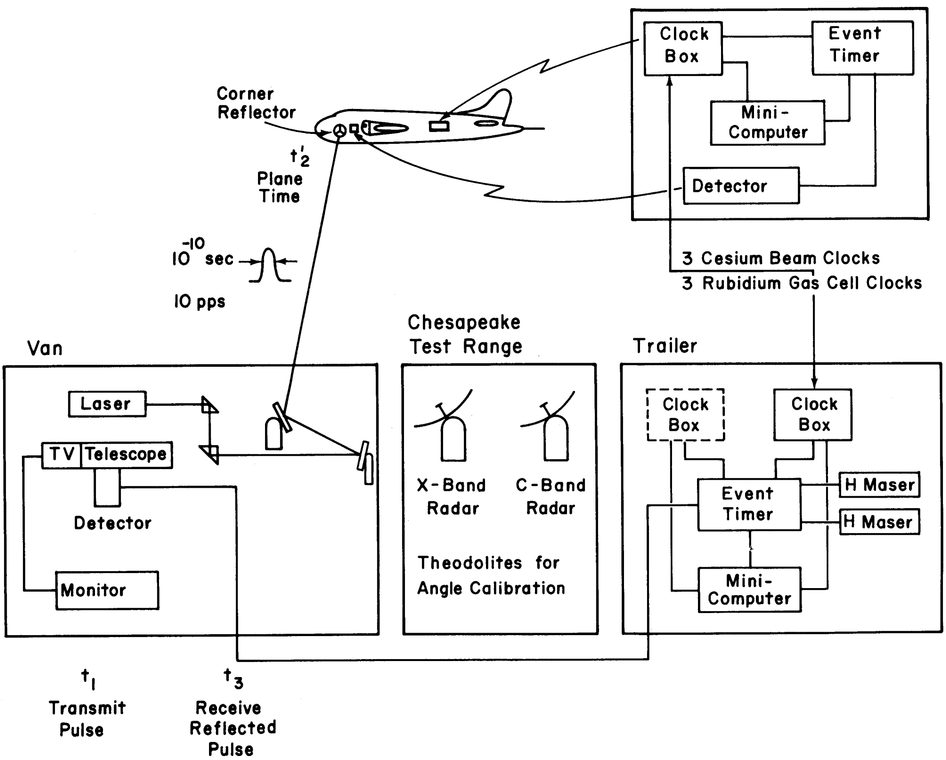


Fig. 22. Schematic diagram of the local flights with laser pulse time comparisons.

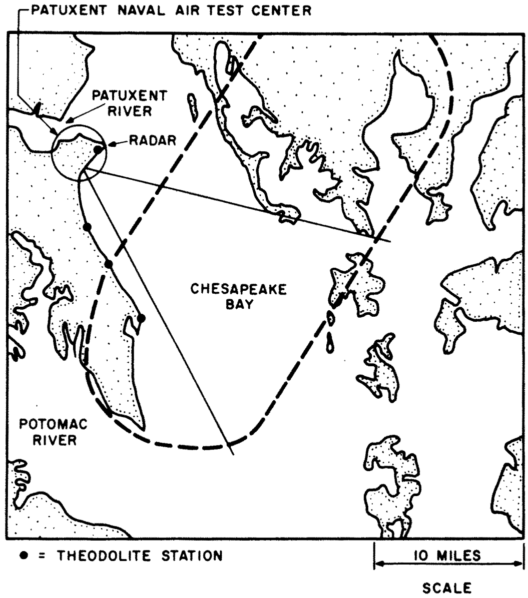


Fig. 24. Approximate path of the aircraft over the Chesapeake Bay.

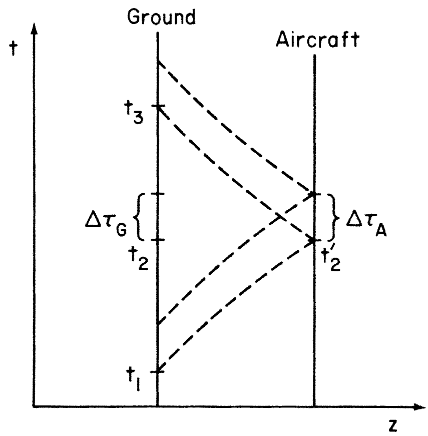


Fig. 25. Comparison of proper time intervals with short light pulses.

light pulses to the transmitter within the angles shown in Figure 24. This equipment allowed the Einstein prescription for remote time comparison to be carried out.

On the ground there was an identical clock box containing three cesium and three rubidium clocks, and also a similar photodetector, event timer, and minicomputer. The laser produced pulses with an energy of 0.5 millijoules, 0.1 nanosecond in duration (3 cm long), at a rate of 10 pulses per second. The light from the laser was directed to the plane by a mirror whose rates in azimuth and elevation could be controlled. The transverse width of the light pulse at the distance of the place was about one-half the length of the plane. The reflected laser pulse was received in a 19 cm diameter refracting telescope to which was also coupled a closed circuit television system for observing the plane.

The plane was tracked continuously by both x-Band and C-Band radars from the Chesapeake Test Range as it circled over the Chesapeake Bay for 15 hours in an approximate race track pattern which is shown in Figure 24. The radars provided the information on altitude, velocity, and position needed to calculate the proper time integral from equation (36) to compare with the directly measured proper time difference between the airborne and ground clock sets.

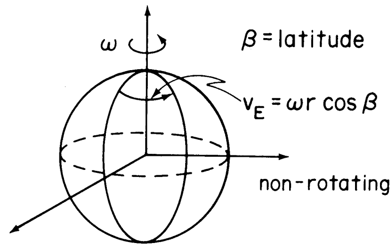


Fig. 26. Inertial system centered on earth used for evaluating the proper time interval.

The appropriate space-time diagram is shown in Figure 25. The Einstein prescription identifies the time t_2, midway between t_1 and t_3, as simultaneous with the reflection/detection time t_2' at the plane. But even though the end points of the proper time intervals $\Delta\tau_G$ and $\Delta\tau_A$ are respectively simultaneous in coordinate time, the elapsed proper time intervals themselves are not equal. The proper time integral to be evaluated is:

$$\tau_A - \tau_G = \int_0^T [(\phi_A - \phi_G)/c^2 - (v_A^2 - v_G^2)/2c^2] dt, \qquad (51)$$

where t represents the coordinate time in the local inertial frame whose origin is attached to the center of the freely-falling earth and which is non-rotating as shown in Figure 26. The time T is the duration of the flight. In this frame, the vector velocity of the aircraft is:

$$\vec{v}_A = \vec{v}_A{}^* + \vec{\omega} \times \vec{r}_A, \qquad (52)$$

where $\vec{v}_A{}^*$ is the velocity of the aircraft with respect to the surface of the earth, $\vec{\omega}$ is the angular velocity of the earth, and $\vec{r}_A$ is the radius vector to the aircraft. When one squares $\vec{v}_A$, there is a cross product term

$$2\vec{v}_A{}^* \cdot (\vec{\omega} \times \vec{r}_A) = 2(\vec{v}_A{}^*)_{East}\, \omega\, r_A \cos\beta, \qquad (53)$$

where β is the latitude. $(\vec{v}_A{}^*)_{East}$, the eastward component of the aircraft velocity, will be positive for eastward motion and negative for westward motion, giving the asymmetry in the round the world flights we described earlier. In our local flights the integrated effects of this term were too small to be measured, but it was very significant in the global experiments to be described later. In evaluating equation (51) it was just barely necessary to expand the earth gravitational potential ϕ to the quadrupole moment term.

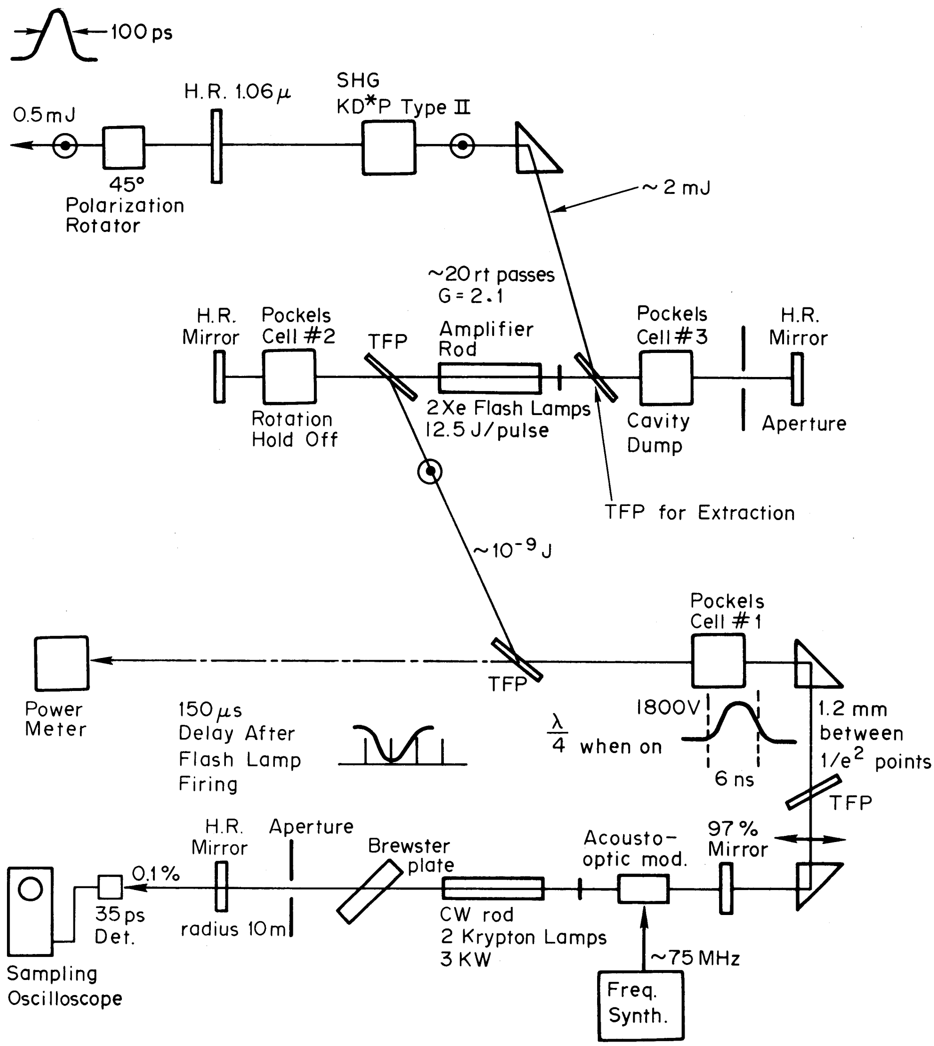


Fig. 27. Diagram of the neodymium YAG laser with mode-locking, multi-pass amplification, and frequency doubling.

The Short Pulse Laser System

The quantum optics in the experiment mainly involved the mode-locked oscillator, multi-pass amplifier, neodymium-doped yttrium aluminum garnet (YAG) laser. Its output pulse at 1.06 microns wavelength was doubled in frequency by a KD*P non-linear crystal to a wavelength of 0.53 microns (green light). A diagram of the laser is shown in Figure 27. The short pulses of 0.1 ns duration are produced by the acousto-optic modulator in the oscillator cavity at the bottom of the diagram. The modulator acts as a switch, opening at the zero-crossing times of the 75 MHz standing acoustic wave set up in a piece of quartz. (At other times light is diffracted from the density variations -- the Debye-Sears effect). The optical gain in the continuously pumped laser rod sustains a short pulse that is reflected back and forth in the laser cavity, whose length is adjusted so that the pulse arrives at the acousto-optic modulator in synchronization with the zero crossing times. The train of pulses leaving the 97% reflecting mirror may be regarded as resulting from the coherent superposition of the many longitudinal cavity modes within the frequency width of the Nd YAG laser crystal -- hence the terminology, mode-locking.

One of the pulses in the train is selected by applying a short pulse of voltage to a Pockels cell which results in the horizontal polarization being rotated to the vertical. The thin film polarizer (TFP) then reflects this pulse, which has an energy of about 10^{-9} joule, into a multi-pass amplifying cavity by means of another thin film polarizer (these are Brewster angle devices). Pockels cell #2 rotates the polarization of the pulse back to the horizontal during the double pass, and is then shut off, so that the pulse passes through the TFP. After about 20 round trip passes through the amplifier, the pulse energy reaches about 2 millijoules and the pulse is switched out by activating Pockels cell #3 to rotate the polarization so that the extraction TFP reflects the pulse. The green light is produced by non-linear electron oscillations in the KD*P Type II second harmonic generating crystal. The high reflectivity filter at 1.06 microns suppresses the un-doubled energy and the result is about 0.5 millijoules of green light in a pulse of 100 picoseconds, which can be seen and detected with high efficiency.

We selected a pulse 10 times per second for the time transfer measurements. This rate meant that the laser beam need hit the reflector/detector combination for only a few seconds to accumulate the 25 measurements we deemed necessary for averaging purposes for a single time comparison. This was a small enough number so that the recorded epochs could be read to the ground over the radio link after the laser firing was completed. This allowed accurate time comparisons during the flight using the Einstein prescription.

Cesium Beam Atomic Clocks

The way atomic clocks work should be described briefly, particularly since the type which performed best for us uses the Stern-Gerlach effect for state selection which has been much discussed during the talks on quantum mechanical measurement

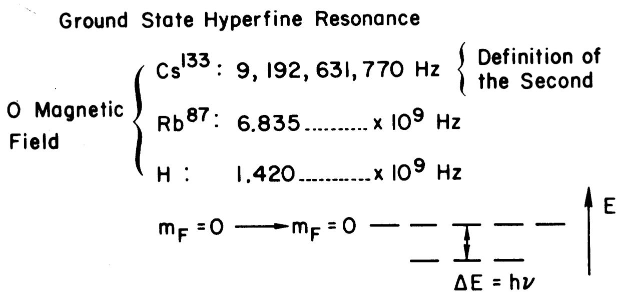


Fig. 28. Frequencies used in modern atomic clocks.

theory at this Advanced Study Institute.* There are three currently useful types of atomic clocks: the cesium atomic beam type, which derives from work by O. Stern, W. Gerlach, I. I. Rabi, J. R. Zacharias, and N. F. Ramsey; the rubidium vapor gas cell type using optical pumping and optical detection, which was proposed by R. H. Dicke and which was developed initially at Princeton, notably by T. R. Carver, with contributions by others, including the present writer; and the hydrogen maser proposed by N. F. Ramsey and developed initially at Harvard by D. Kleppner and M. Goldenberg and by R. F. C. Vessot of the Smithsonian Astrophysical Observatory.

All of these utilize a ground state hyperfine transition at microwave frequency. These are listed in Figure 28 for the case of zero magnetic field. The ground hyperfine states are shown for Rb^{87} whose nuclear spin of 3/2 combines with the electron spin of 1/2 to give total angular momentum states F=2 and F=1. The $M_F = 0 \rightarrow M_F = 0$ transition is used because its frequency is influenced by magnetic fields only in second order. The definition of the second is in terms of the number of oscillations of the cesium frequency shown in Figure 28.

---

*It should be noted that there is a genuine historical connection between the Stern-Gerlach experiment and General Relativity.[21] In 1920, public interest in General Relativity was intense following the successful confirmation of Einstein's prediction of the gravitational deflection of starlight by the 1919 solar eclipse expeditions organized by Sir Arthur Eddington. To satify this interest, Max Born, who was then chairman of the Physics Department at the University of Frankfurt-am-Main, gave a series of public lectures on relativity for which admission was charged and which were very well attended. The money so obtained was used during the early 1920's to support at Frankfurt the experimental research of Otto Stern and Walter Gerlach!

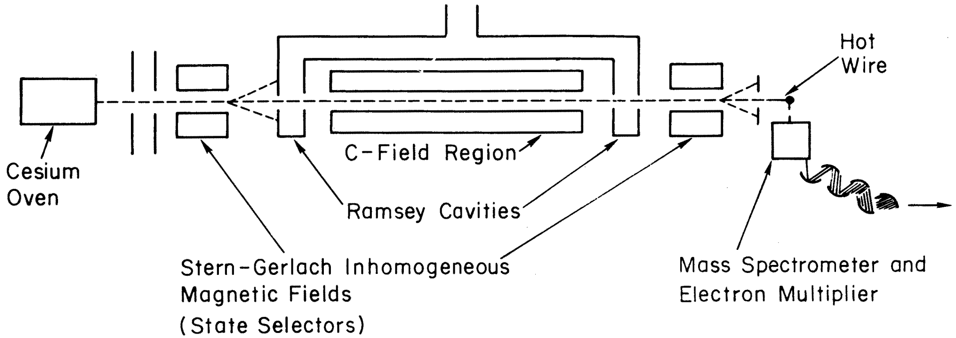


Fig. 29. Physics of the cesium beam atomic clock.

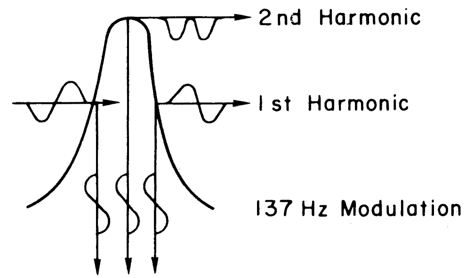


Fig. 32. Frequency modulation to determine sense of error.

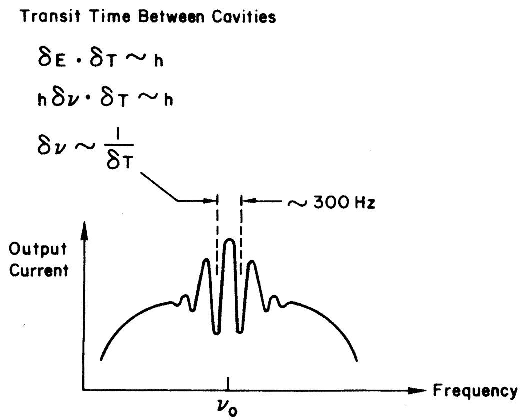


Fig. 30. Resonance from the Ramsey method.

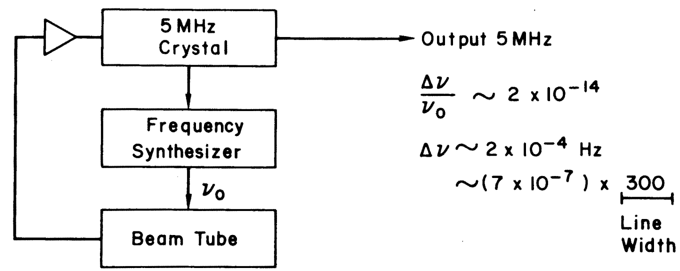


Fig. 31. Feedback from the beam tube to stabilize the 5 MHz oscillator.

Let us briefly examine the physics of the cesium beam clock by referring to Figure 29. The major parts of the cesium beam shown are in a high vacuum enclosure, which is itself surrounded by triple magnetic shielding. (We are describing the version engineered by the Hewlett-Packard Company). The oven on the left heats the cesium to produce an atomic beam (actually a ribbon) defined by apertures. On passing through the Stern-Gerlach state selector, atoms in one of the $M_F = 0$ states are deflected through the first microwave cavity, and atoms in the other (F, M_F) states are blocked. On leaving the first cavity, the atoms proceed through a region of uniform magnetic field (the "C-field") and then through the second microwave cavity (actually the two cavities are part of the single U-shaped cavity as shown). If the frequency of the microwaves in the cavity matches the hyperfine transition frequency, transitions to the other $M_F=0$ state are induced and the second Stern-Gerlach selector deflects the atoms onto a hot wire which ionizes them. The ions are sent through a mass spectrometer to reduce noise from impurity atoms and then into an electron multiplier to produce a current pulse for each atom which has successfully run the "obstacle course". The apparatus is adjusted so that the maximum number of atoms get through when the microwave frequency in the U-shaped cavity exactly matches the hyperfine transition frequency.

The output current as a function of frequency ν is plotted in Figure 30. The separated cavities produce a sort of interference effect since the electromagnetic field oscillations in each are coherent. This produces the oscillations shown with a narrow central peak. The width of this peak $\delta\nu$ is related to the transit time δT between the separated cavities according to the uncertainty principle. The idea was due to N. F. Ramsey. For the size of the H-P tubes, (about 25 cm) the width is about 300 Hz.

How is such a device made into a clock? One synthesizes electronically the microwave resonance frequency ν_0 (= 9.192631770 GHz) from a very good crystal oscillator at 5 MHz as shown schematically in Figure 31. A frequency modulation at 137 Hz (no connection with the fine structure constant!) is imposed on ν_0 as sketched in Figure 32. If the synthesized frequency is exactly ν_0, then the 137 Hz modulation produces a second harmonic in the detection current at the peak of the Ramsey resonance. If the frequency is $> \nu_0$, then the modulation produces a first harmonic (i.e. fundamental) frequency in the beam current. If the frequency is $< \nu_0$, the first harmonic is of the opposite phase as shown in Figure 32. This change of phase allows the sense of the frequency error to be measured and used in a feedback circuit to correct the 5 MHz crystal frequency as shown in Figure 31. The electronic engineering in the HP clocks is outstanding. When we controlled the environment of the clocks in the way described later, it was possible to achieve a frequency stability

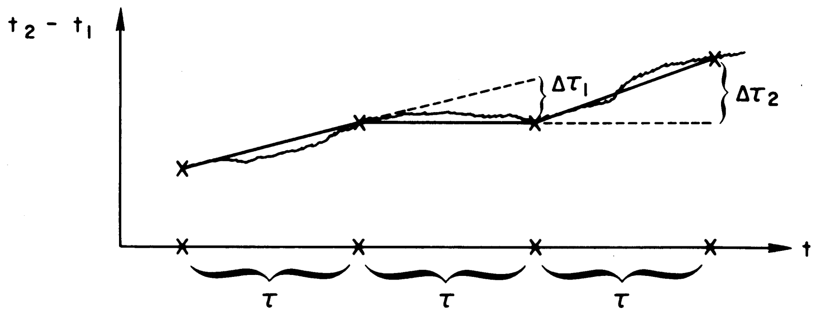


Fig. 33. Illustrating the definition of the Allan variance $(\Delta\tau)_{rms}/\tau \equiv \sigma(2;\tau)$ used to describe the behavior of two atomic clocks.

$\Delta\nu/\nu_0 \approx 2 \times 10^{-14}$, over a day. This corresponds to splitting the 300 Hz resonance by 7×10^{-7} as shown in Figure 31.

How does one measure such performance? Clearly one must compare a clock with a set of similar clocks. We measure the phase differences of the 5 MHz signals from the clocks (differences in zero-crossing times). The rates of clocks are always slightly different. This does not matter for many purposes as long as the relative rates are stable. There is always a certain amount of irreducible fluctuation in the time difference between two clocks. In the case of cesium beam clocks it is due to the random walk effects associated with counting the atoms as they come through the "obstacle course." It is the practice in the time keeping community to characterize clock performance statistically as shown in Figure 33. The variations in the time difference of the two clocks is plotted as a function of time. One chooses a time interval τ and constructs a series of chords of this duration

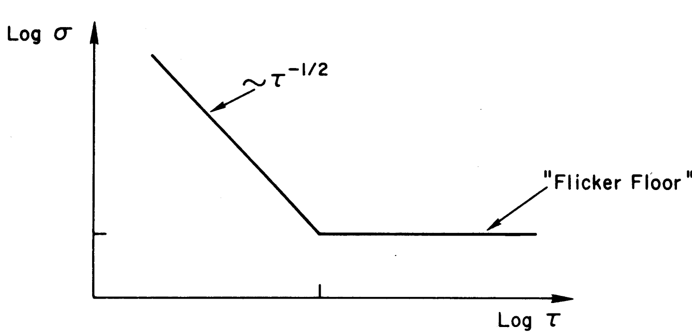


Fig. 34. Illustrating the occurance of a lower bound to the fractional stability of clocks.

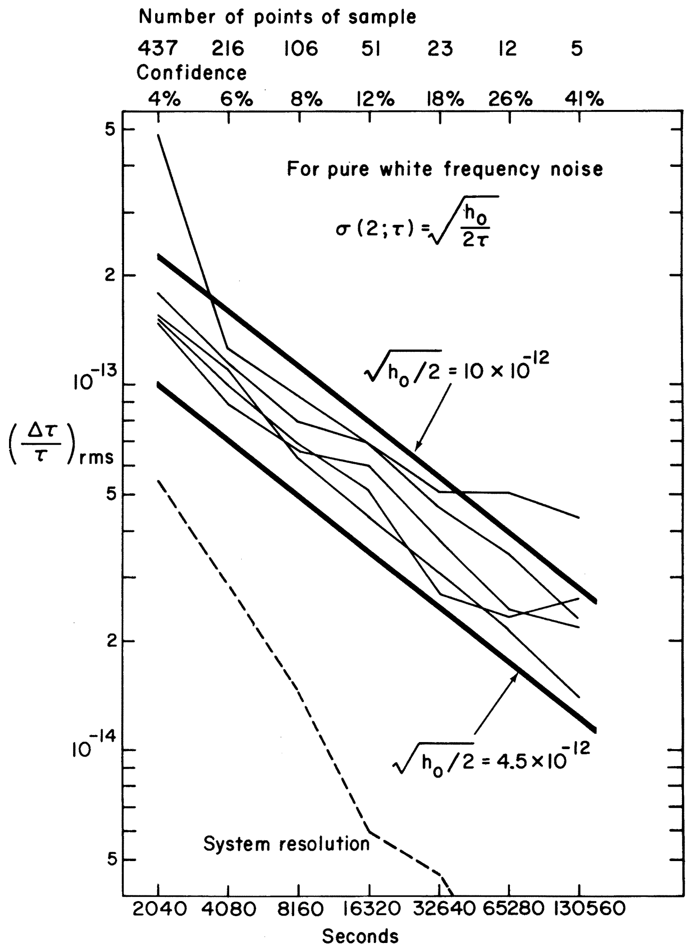


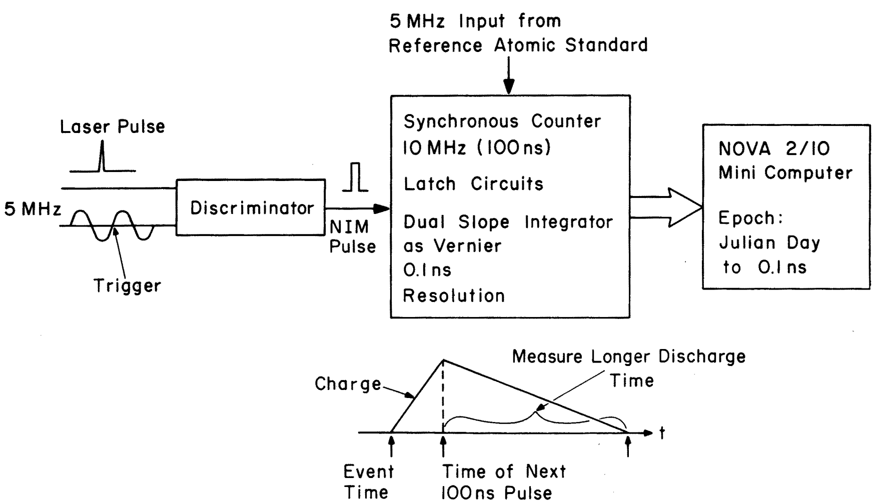
Fig. 35. Measured fractional stability of the cesium beam clocks used in the experiment.

joining actual data points. The differences, called $\Delta\tau_i$ in the diagram, between the projected chord and the next actual point after the interval τ are calculated. The variance $(\Delta\tau)_{rms}/\tau$ is given the symbol $\sigma(2;\tau)$ as shown and is named after D. W. Allan of the National Bureau of Standards who introduced it. For white frequency noise, the Allan variance is expected to vary with τ as $\tau^{-1/2}$, as shown in Figure 34. All clocks with which we have experience reach a minimum value of σ as τ is increased, usually called the "flicker floor."

Actual measurements of $\sigma(2;\tau)$ for several values of τ on a log-log plot are shown in Figure 35 for five of the cesium clocks used in the measurements, referred to a sixth cesium clock as reference. It is clear that the predicted behavior for white

BOX 1. Converting an Atomic Frequency Standard into a Clock.

In the text there is frequent use of the phrase "atomic clock" in referring to what is perhaps more properly termed an atomic frequency standard. To measure elapsed time, an event timer, shown schematically below, is associated with a reference frequency standard. It establishes a time scale with respect to a selected origin event by using synchronous counter and latch circuits to count the number of 100 ns periods of the frequency doubled 5 MHz input occurring between the origin event and the event whose epoch is to be measured. Resolution of 0.1 ns is achieved by using a dual slope integrator, as sketched, to measure the number of periods of an internal 80 MHz oscillator during the longer discharge time of a capacitor which is related to the charge time by a factor of 125. A minicomputer controls the event timer and stores the epoch of the measured events. The event timer was originally designed and built for lunar laser ranging measurements, so the epoch is measured in Julian Days.



BOX 2. "...a clock is an essentially nonmicroscopic object." (Eugene Wigner)

The limitations on the accuracy of a clock imposed by quantum mechanics have been analyzed by Professor Eugene Wigner and discussed in his retiring presidential address to the American Physical Society, titled "Relativistic Invariance and Quantum Phenomena". (This was published in the Reviews of Modern Physics, vol. 29, No. 3, July, 1957 and is also reprinted in the collection of scientific essays by Professor Wigner, Symmetries and Reflections, Indiana University Press, 1967.) One of his important conclusions is that

> "...the inherent limitations on the accuracy of a clock of given weight and size, which should run for a period of a certain length, are quite severe."

leading to his statement in the title of this Box. He developed a formula for the mass based on a possible (but somewhat impractical) model of a clock, which he recognized as producing only the "best present estimate":

$$m > n^3 \hbar t/\ell^2,$$

where

$\hbar$ = Planck's constant $/2\pi$
ℓ = linear dimension
t = resolution of the clock
$n = T/t$ where T is the running time of the clock.

It is interesting to evaluate this expression for the cesium frequency standard/event timer/ computer combination discussed in Box 1:

$$\left.\begin{array}{l} t = 0.1 \text{ ns} \\ \ell \approx 1 \text{ meter} \\ T = 5 \text{ days.} \end{array}\right\} \Longrightarrow \quad m > 700 \text{ kg}$$

The combination above has a mass of about 100 kg, but when one includes the massive aluminum box used for environmental control of the ensemble of clocks, racks for carrying the computer and event timer, power supplies and other supporting equipment, the total mass exceeded 700 kg. In fact, the Navy and Air Force crew members, who so ably supported the experiments, called our clock assembly "the Two-Ton Timex."

frequency noise is observed, with individual differences among clocks. The value of $(\Delta\tau)_{rms}/\tau$ of 2 or 3 x 10^{-14} for a period of one day (86,400 seconds) which was achieved for most of the clocks amounts to approximately 2 or 3 nano seconds per day. This excellent performance in commercially available clocks was produced by making several modifications to them in addition to controlling their environment very carefully. The beam current was increased by a factor of 2 (at the expense of beam lifetime); additional integration was added to the quartz crystal control loop to eliminate the frequency offset that would be produced by steps or ramps in the crystal oscillator frequency; and a proprietary modification was made that is now standard for all Hewlett-Packard beam tubes. By careful packaging of the clocks for environmental control, we were able to maintain this performance during actual aircraft flights. However, a flicker floor was encountered at $\sigma \approx 10^{-14}$ for $\tau \approx 7$ days , even with these controls. Its specific origin is unknown.

Some Significant Features of the Experiments

Ensemble of Clocks. An ensemble of clocks, not just one clock, was used both in the aircraft and on the ground. Members of each ensemble were intercompared with each other every 200 seconds. In this way an average or "paper clock" time scale can be constructed by the computer and individual clock readings compared with the average. This procedure provides some reduction of statistical errors and also a means of identifying a rate change of a clock, if this should occur. Figure 36 shows six cesium beam clocks being adjusted by Leonard Cutler . Three would go into the flying clock box and three into the ground clock box (The cesium beam tube is the cylindrical object at the bottom of the left-most clock.)

Environmental Control. Very careful environmental control of the clocks was maintained. The idea was to keep changes in temperature, pressure, magnetic fields, voltages, and mechanical motion sufficiently small so that the previously measured systematic changes in clock rate from these causes were kept less than 10^{-14}. There would then be no need for systematic corrections since the intrinsic clock stability over the pre-flight, flight, and post-flight period was about 2×10^{-14}. We will not give details here (see references 1, 19 and 20). A temperature controlled air flow system was used for each clock. A temperature gradient of about 10°K existed between the entrance and exit temperatures of the air for heat transfer, but the temperature change at points within a clock (monitored by thermistors) was kept to less than 0.06°K. A hermetically sealed thick-walled aluminum box housed the clocks and was kept at a

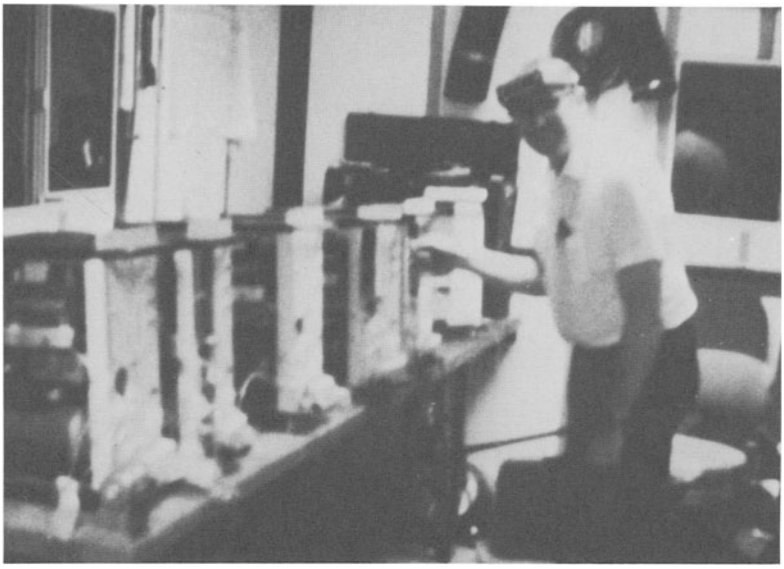


Fig. 36. Six Hewlett-Packard cesium beam clocks being adjusted by Leonard Cutler.

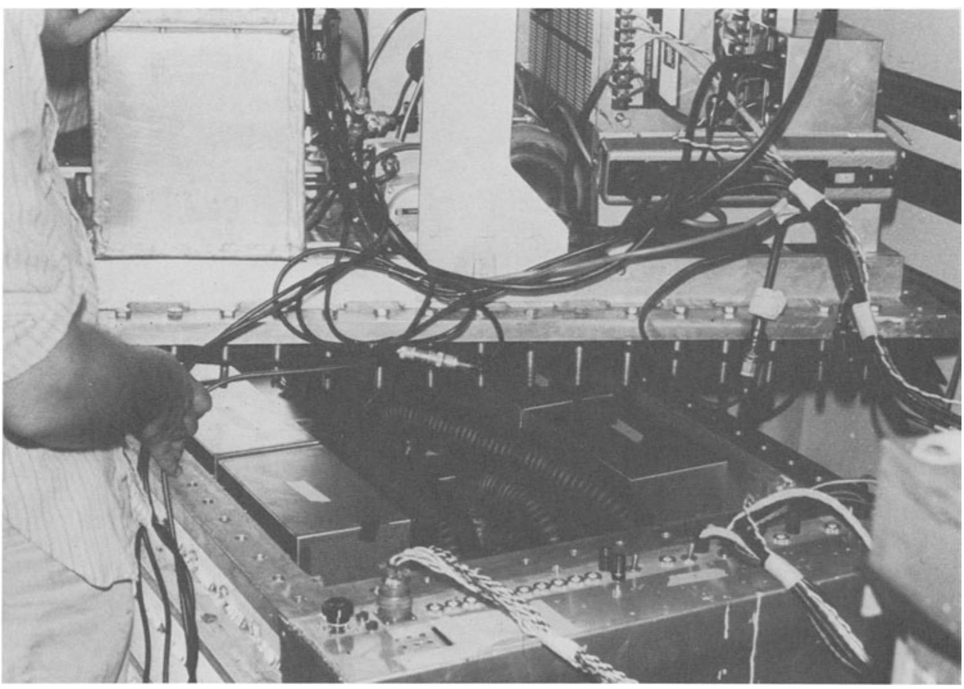


Fig. 37. A "clock box" used to shield the clocks from environmental changes.

pressure slightly greater than ground level atmospheric pressure. Pressure changes were kept to less than 1 Torr. A magnetic shield of 60 mil thick Mo-Permalloy surrounded each clock to prevent changes in the direction of the earth's magnetic field, caused by the changing orientation of the plane, from affecting its rate. Vibration and shock isolation was achieved by mounting the clock box on pneumatic mounts. The resonant frequency was about 3 Hz and the characteristic frequencies of the aircraft were about 80 Hz. Almost critical damping was achieved by using expansion cylinders following an adjustable orifice, providing an increase of isolation of 12 db per octave with increasing frequency. The clock box, with its lid about to be placed on, is shown in Figure 37. The air flow hoses leading to the magnetic shields can be seen. The pressure control and voltage regulation equipment is mounted on the lid. The box on the left of the lid contained three small rubidium optically pumped clocks made by Efratom, Inc. Figure 38 shows the clock box mounted in the P3C aircraft during a flight.

Returned Clocks. The use of an aircraft to elevate a clock ensemble in the earth's gravitational field allowed the direct comparison of the airborne and ground clock sets for a period before the flights and the return of the airborne clocks for a period of direct comparison after the flights. The relative rates of the two clock sets before and after the 15 hour flights could thus be compared, in addition to measuring the difference in elapsed proper times. This difference was determined by projecting clock comparison data forward from the pre-flight monitoring period and backward from the post-flight monitoring period, making the lengths of the projections proportional to the duration of the monitoring periods. No rate changes were observed beyond the statistical expectations for stationary clocks. The use of an aircraft also allowed repeated flights with the same clocks. There were five 15 hour flights, with clock box 1 flying three times and clock box 2 twice. This gave some check on possible systematic errors. Before the long flights were undertaken there were five test flights, each of about two hours duration.* These allowed us to identify problems and to improve the performance of the experiment. The short distance between the aircraft and the ground clock set during the ground comparison periods is shown in Figure 39. The ground clocks and computer equipment were in the 40 foot trailer between the plane and the hangar. The bus at the end of the hangar contained the laser and beam directing optics, receiving telescope, and photodectector.

* It was a moment of great satisfaction for the experimenters when an unequivocal time difference of 6ns ± 1ns was observed after the first test flight in May, 1975.

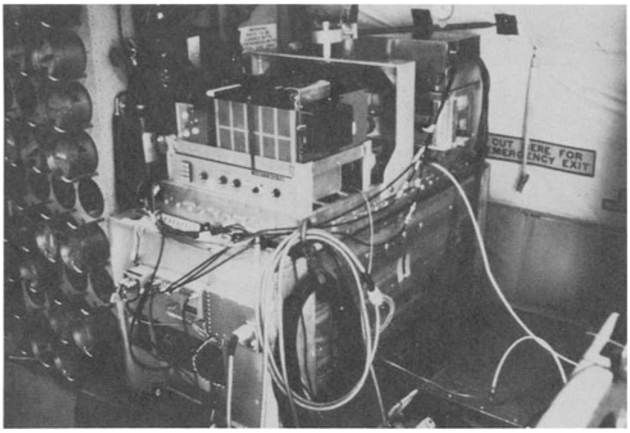


Fig. 38. Clock box mounted in the P3C aircraft.

<u>Laser Pulse Remote Time Comparison</u>. The experimental realization (which we believe to be the first) of Einstein's 1905 prescription of using reflected light pulses to compare clocks at a distance, allowed the measurement of the continuously developing proper time difference during the flights. (The results exclude any possibility of discontinous jumps in the time difference, as has occasionally been suggested by people who question some of the ideas of relativity.) The technique has very great technical advantages in addition to its conceptual clarity. It offers both the most precise and the most accurate method known for practical comparison of distant clocks. Its relative freedom from atmospheric delay effects was explained in the first part of this paper. In addition, there are no Doppler effect complications even when the reflections are from a moving aircraft. One actually compares simultaneous events between the ground and

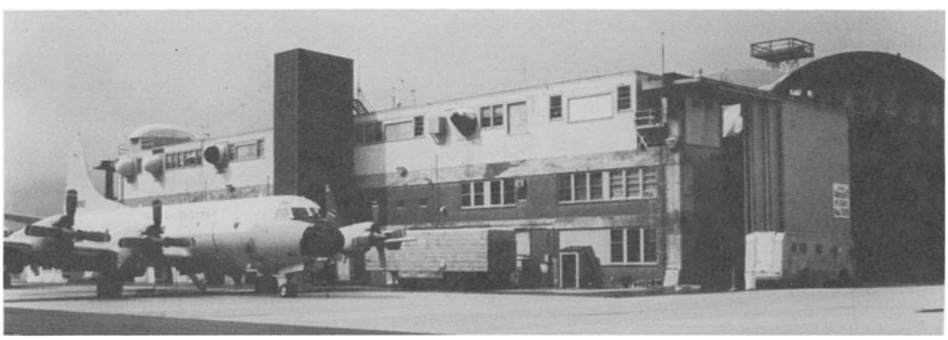


Fig. 39. P3C aircraft next to the trailer containing the ground clocks beside a hangar at the Patuxent Naval Air Test Center.

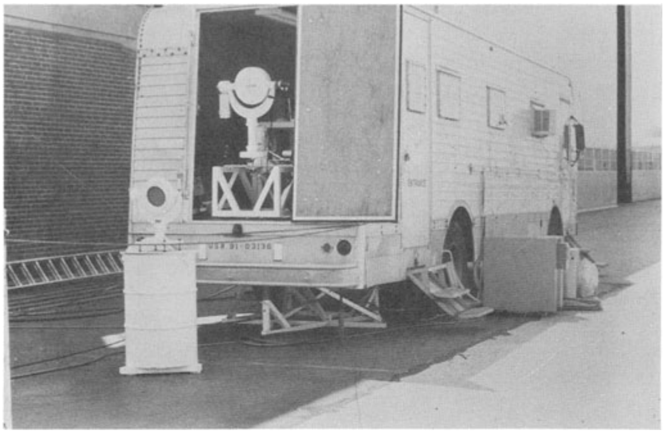


Fig. 40. Laser beam directing optics and bus containing laser, receiving telescope and photodetector.

aircraft clocks and the relative velocity does not enter the comparison. Figure 40 is a picture of the rear of the bus showing the directing mirrors for the laser beam. Inside the bus was the short pulse laser decribed earlier and the 19 cm receiving telescope which passes light through a beam splitter to a fast photomultiplier detector and to a closed circuit television camera. The flights were conducted largely during the hours of darkness so that the plane could be more easily tracked by its two landing lights. When the laser beam with its divergence of 0.5 milliradian was hitting the corner reflector, a third spot of

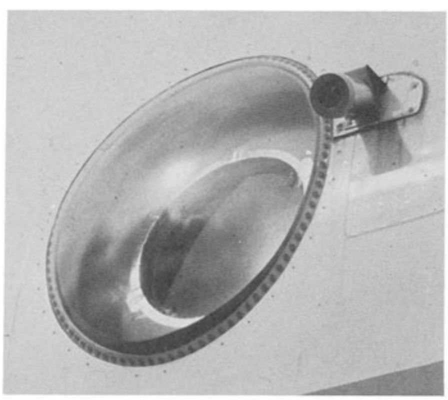


Fig. 41. Optical corner reflector next to a forward observing window.

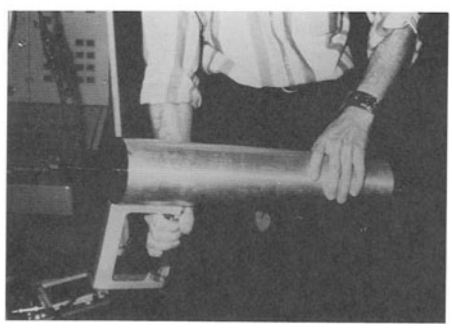


Fig. 42. Detector for laser light pulses on the aircraft.

light would appear on the tracking monitor. On the plane, the 0.5 millijoule per pulse, 10 pulse per second laser beam would enter the plane through the observation windows and actually cast shadows. Figure 41 is a picture of one of the lunar type corner reflectors mounted at a downward angle next to a forward, observing window on the P3C aircraft. A fast RCA 31024 photomultiplier detector in a cylindrical housing with neutral density filters to attenuate the light pulses, shown in Figure 42, was held just behind this window. It was necessary to allow for the slant distance along the laser beam between the corner reflector and the detector in making the time transfer calculations. Using the average of 25 pulses, described earlier, a precision of 0.2 to 0.3 nanoseconds was achieved in the laser time comparison.*

Experimental Results from the Local Flights

The laser pulse time comparison data from a 15 hour flight on November 22, 1975 is plotted in Figure 43. The ordinate is the time difference between the "paper clocks" of the airborne and ground clock sets. The abscissa is the elapsed time between the start and finish of the measurements. (The 12 hour gap in data during the pre-flight period represented the need of the laser operator to get some rest before the flight and post-flight period.) The flight occurred between 24 and 39 hours on the abscissa. The change in relative rate is very apparent: from about - 30ns/day to about + 45ns/day during the flight, and back to - 30ns/day after the flight.

If the relative rate between the clock sets as measured before the flight is subtracted out, the result is as displayed in Figure 44. The return of the relative rate after the flight to the pre-flight value is clearly shown. Also plotted in this Figure is the result of the direct comparison of the two clock sets by measurement of the relative phases of the 5 MHz clock outputs. This is shown by the irregular solid line which exhibits the natural statistical fluctuations of the cesium atomic beam clocks discussed earlier. The agreement of the measured time difference with the predicted value of 47 ns is seen to be very close.

It is interesting to plot the laser time comparison points during the flight on an expanded scale as is done in Figure 45. The data is fit by three linear segments, which are projected by

* Moving pictures of the aircraft flights and laser pulse time transfer are included in the BBC television production "Einstein's Universe", written by Nigel Calder, and shown at the time of the Einstein Centennial in 1979 and on several occasions since then.

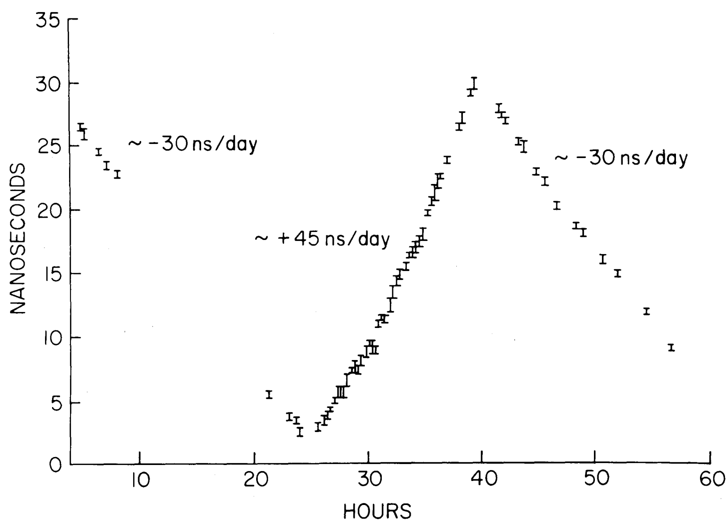


Fig. 43. Laser pulse time comparison data for the flight on November 22, 1975.

dashed lines in order to display the changes in slope. These slope changes are produced by the changes in gravitational potential accompanying step changes in the altitude of the aircraft. In order to stay aloft for 15 hours, a heavy load of fuel must be carried, which prevents the plane from initially climbing above 25000 feet. It circled at that altitude for five

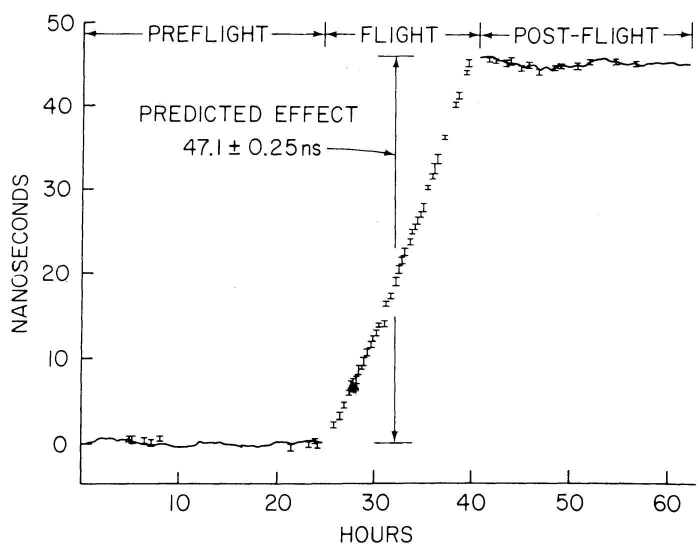


Fig. 44. Data from Fig. 43 with the preflight relative rate subtracted.

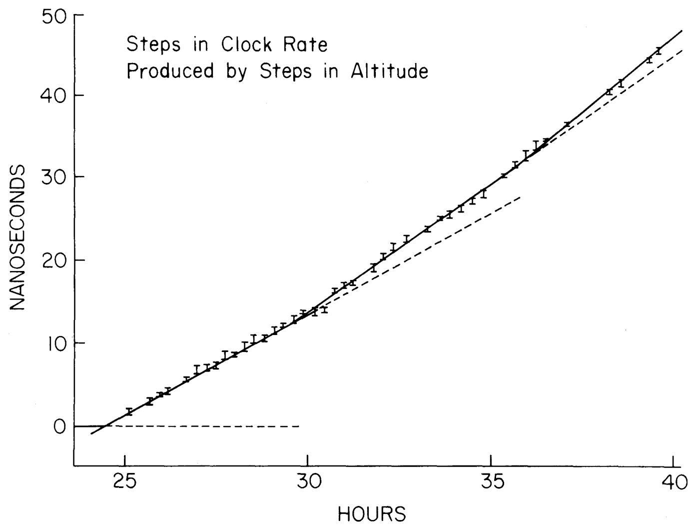


Fig. 45. The atomic clock as an altimeter.

hours, burning up fuel, and was then able to climb to 30,000 feet, where it stayed again for five hours before climbing to 35,000 feet for the final five hours. This was standard procedure, but it furnished a very convenient modulation of the gravitational potential for our measurements.

This modulation is shown in the top curve of Figure 46, which is a plot of the change in gravitational potential of the airborne clocks (divided by c^2) during the flight as calculated from the radar tracking data. The steps in altitude are very apparent. Also plotted on the same axes is the quantity $-v^2/2c^2$, the motional contribution to the proper time resulting from the spatial part of the metric. The periodic form of the graph is due to the modulation by winds of the velocity of the plane relative to the ground as it executed its repeated racetrack pattern shown in Figure 24.

The results of integrating equation (51) are plotted in Figure 47. In addition to the net effect, the gravitational potential and velocity contributions are integrated separately. Also the laser pulse time comparisons are superimposed on the integral of the net effect. The oscillations in $-v^2/2c^2$ are too short to produce any large effect, but the average value of the velocity term does produce a significant effect, about 11% of the potential effect and about 12% of the net effect. We deliberately operated the plane at as low a speed as possible in order to maximize the potential effect since the accurate measurement of this was the main objective of the experiments. However, the

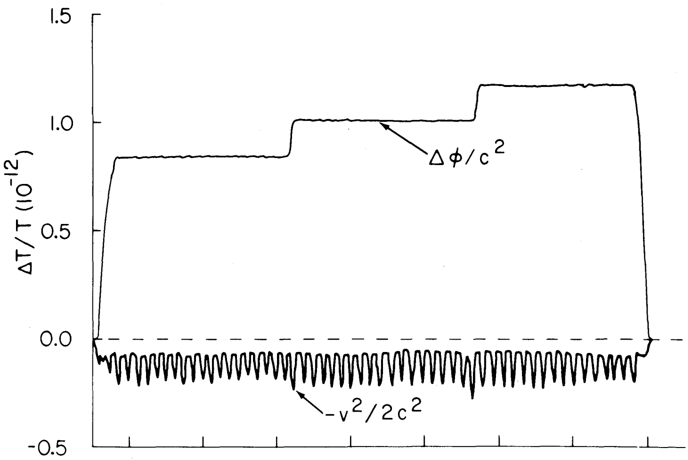


Fig. 46. Radar tracking data converted to $\Delta\phi/c^2$ and $-v^2/2c^2$.

plane could not fly safely at a speed less than about 200 knots.

The overall accuracy of measurement from the five flights of about 1.5% therefore yielded also a measurement of the velocity effect for macroscopic clocks with an uncertainty of approximately 10%.

Another result of the experiments is a clear record that clocks in the airborne set maintained the same rates with respect to each other while flying as on the ground. In Figure 48 is shown the time difference between flying clock #3 and the average

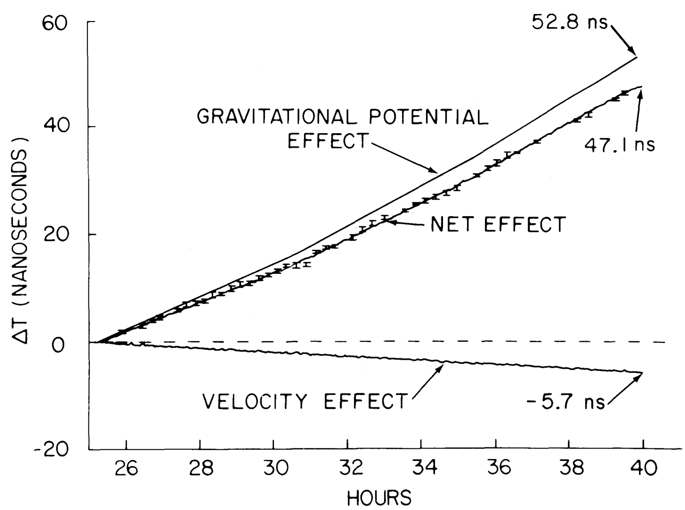


Fig. 47. Growth of the proper time integral during flight.

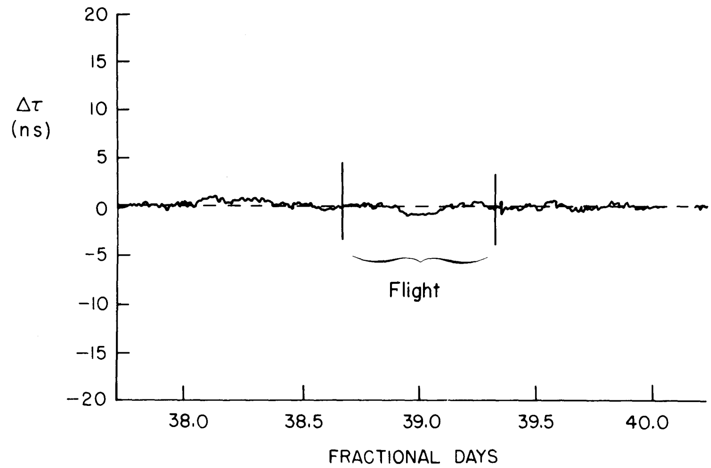


Fig. 48. Comparison of Flying Clock #3 to the average of all flying clocks shows no discernable effect of flight.

of the airborne set, with the beginning and end of the flight indicated. There is no discernable effect caused by the flight. Again, the fluctuations are characteristic of the performance of the cesium beam clocks when they are compared with a resolution of 0.1 nanosecond which our event timer provided. (Recall that 1 ns/day ≈ 10^{-14}) To contrast with Figure 48, there is plotted in Figure 49 the comparison of flying clock #3 with the ground paper clock by means of the phase comparison of the 5MHZ signals. The discontinuity is similar to that in Figure 44.

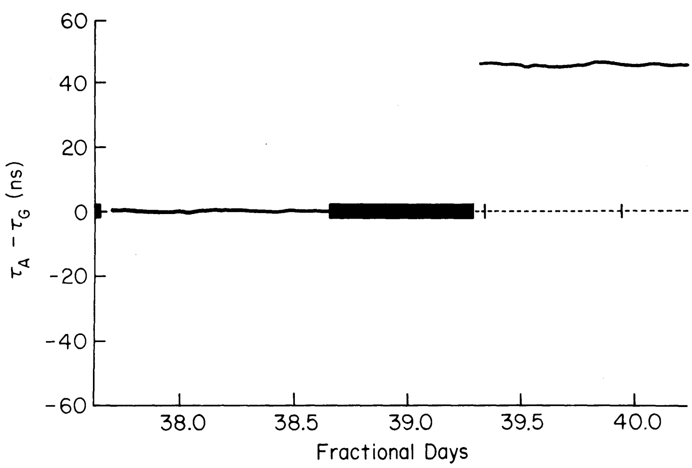


Fig. 49. Comparison of Flying Clock #3 to the average of ground clocks shows relativistic time difference after flight.

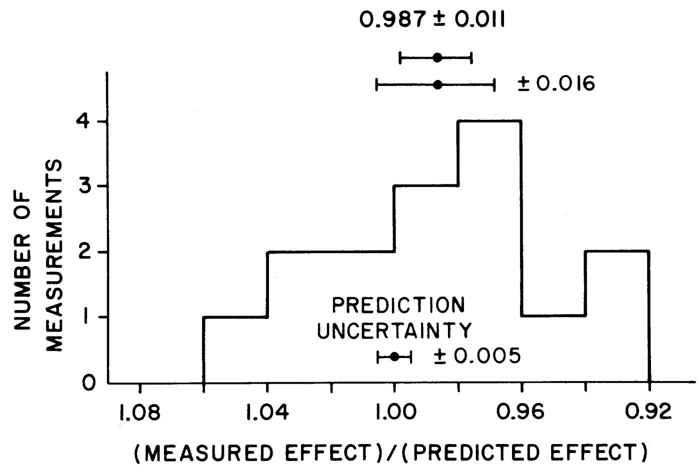


Fig. 50. Histogram of 15 measurements: three cesium beam clocks on each of five flights.

Considering all five flights with three well behaved cesium clocks per flight, a histogram for results of 15 measurements is shown in Figure 50, where the abscissa is the ratio of the measured effect to the predicted effect from the radar tracking data. The prediction uncertainty was only 0.005. Formal statistics yielded 0.987 ± 0.011 for the mean, where the uncertainty is the standard deviation of the mean. To allow for possible systematic uncertainties we are quoting an overall uncertainty of ± 0.016. The ratio is thus $1 - (1.3 \pm 1.6)\times 10^{-2}$.

GLOBAL TRANSPORT OF CLOCKS OVER A LARGE LATITUDE DIFFERENCE

Reconfiguration of Equipment

In late 1976, the Air Force Office of Scientific Research joined the Office of Naval Research and the U.S. Naval Observatory in the support of clock transport experiments over long distances under controlled conditions. A C141 "Starlifter" long range transport aircraft was made available from the 4950th Test Wing at the Wright-Patterson Air Force Base and ground support facilities were provided at the Andrews Air Force Base in Maryland near Washington, D.C. This aircraft is shown on the ground at Andrews in Figure 51.

To provide accurate measurements of latitude, longitude, velocity with respect to the ground, and altitude above the ground for evaluation of the relativistic proper time integral, it was necessary to augment the plane's navigational equipment. Two

Carousel IV inertial navigation systems and a radar altimeter were added, along with automatic digital data logging equipment which recorded a complete set of the measurements on magnetic tape every 0.6 second.

Essentially the same clocks and timing equipment was used as on the earlier local flights with the Navy P3C aircraft, except that the laser pulse time transfer apparatus was not included. To facilitate loading and unloading, the clock box and related equipment were mounted on a large metal frame work which could be attached to a standard Air Force 7 foot by 9 foot cargo pallet. The pneumatic isolation supports for the clock box were improved and the entire framework was enclosed by double wall plywood panels containing insulation. This enclosure could be either heated or cooled, as the conditions in the interior of the plane demanded, to keep the temperature around the clock box reasonably uniform. Figure 52 shows the equipment in its framework and Figure 53 shows the overall enclosure mounted on the cargo pallet, with one wall temporarily removed to provide access to the clock box, in the prefabricated garage structure built at the Andrews Air Force Base to contain the equipment between flights. Only about ten minutes was needed to load or unload the continuously operating clock equipment mounted this way (to be contrasted with the day and a half needed for the P3C). This capability meant that the C141 did not need to stay continuously at Andrews for the clock monitoring periods before and after flights. Figure 54 shows the clock assembly on a fork lift approaching the rear petal doors of the aircraft. Figure 55 shows the assembly being pushed to the front of the aircraft along the floor rollers, and Figure 56 shows it installed in the plane during one of the flights.

Much additional equipment had to be transported to ensure continuous operation of the clocks in an adequately controlled plane interior environment at different landing sites, which could be in both hot and cold climates on the same trip. This included

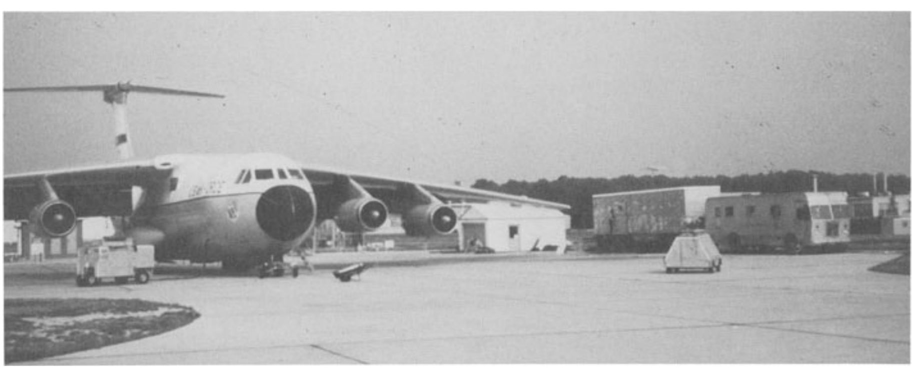


Fig. 51. U.S. Air Force C141 Starlifter on ground at Andrews AFB.

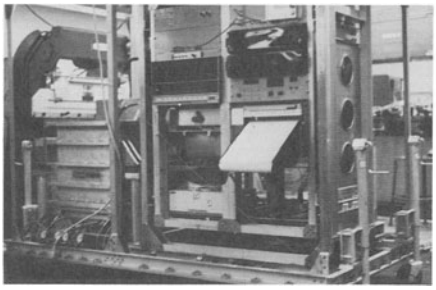


Fig. 52. Clock box and timing electronics being mounted in framework.

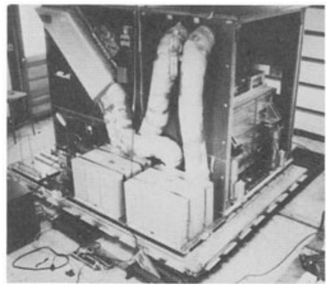


Fig. 53. Clock assembly on cargo pallet with thermal enclosure.

gasoline powered portable electric generators, heaters, and air conditioners, some of it in duplicate. Space was also needed for the fourteen Air Force personnel and eight University of Maryland "clock-watchers" who went on each flight. The entire capacity of the large plane was actually needed.

Independence of Proper Time as a Function of Latitude on the Rotating Oblate Earth

We have discussed earlier "Einstein's Error" in 1905 when he predicted a latitude dependence for proper time because of the changes in surface velocity on the rotating earth as one goes from the equator to the pole. He did not consider the effect of gravitational potential on proper time, discovered only later as a

Fig. 54. Clock assembly being loaded through petal doors of the C141.

Fig. 55. Clock assembly being rolled to front of the C141.

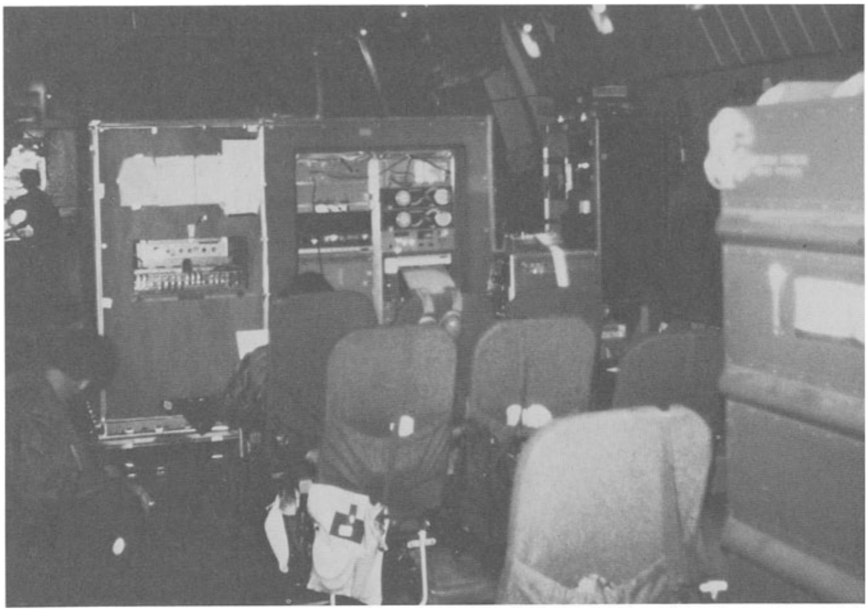


Fig. 56. Installed clock assembly during a flight. The seats are for the travelling "clock-watchers."

consequence of his Principle of Equivalence which he recognized in 1907.

Because of its rotation, the earth has the shape of an oblate spheroid, flattened at the poles. The gravitational potential, in the sense of Newton, is therefore not constant over its surface, but is less at the poles than at the equator. The mean ocean surface (averaged over the periodic tidal deformations) is not a surface of constant gravititional potential, but rather a surface of constant "geopotential" where the centrifugal potential $-v^2/2$, v being the surface velocity due to the rotation, must be added to the gravitational potential,

$$\text{"geopotential"} = \phi - v^2/2. \tag{54}$$

The surface of constant geopotential which coincides with the mean ocean surface is spoken of by geophysicists as the "geoid". Recalling equation (45), which is valid in the Newtonian approximation to Einstein's theory of gravity,

$$d\tau = \left[1 + c^{-2}(\phi - v^2/c)\right]dt,$$

it is clear that proper time has a constant relation to coordinate time along the geoid , and will thus be independent of latitude. (We are, of course, neglecting the small variations of ϕ produced by inhomogeneities of the earth and by departures in the shape of the earth from the ideal oblate spheroid.)

There is a more fundamental way of viewing the "geopotential". If we express the metric of General Relativity in a non-rotating frame of reference, with origin at the center of the earth, in the Newtonian approximation,

$$ds^2 = (1 + 2\phi/c^2)\, c^2dt^2 - dr^2 - r^2d\theta^2 - dz^2 \qquad (55)$$

with cylindrical coordinates,

r = distance from spin axis
z = distance above the equatorial plane,
θ = azimuthal angle.

The transformation to coordinates rotating with the earth is accomplished by

$$r \to r \qquad (56)$$
$$z \to z$$
$$\theta \to \omega t + \theta$$

where ω is the angular velocity of the earth. Then at a fixed spot on the surface of the earth, the metric becomes

$$ds^2 = c^2dt^2 = (1 + 2\phi/c^2 - r^2\omega^2/c^2)\, c^2dt^2. \qquad (57)$$

Therefore, in the coordinate system rotating with the earth, the quantity $\phi - r^2\omega^2/2$ which is just $\phi - v^2/2$ can be regarded, from Einstein's point of view, as a gravitational potential.

<u>Experimental Comparison Between Washington, D.C. and Thule, Greenland</u>

An achievable approximation to an equator to pole transport was from the Andrews AFB (latitude 38°49') to the Air Force Base at Thule, Greenland (latitude 76°32'). After several days of comparison with the ground clock set, the airborne clock assembly was transported to Thule on 23 June, 1977 in a six hour flight. The clocks were kept at Thule for four days and returned on 27 June to the Andrews AFB for a period of comparison with the clocks which had remained there. The results were:

Measured: $\tau_A - \tau_G = 38$ ns $\pm$ 5 ns
Calculated: $\tau_A - \tau_G = 35$ ns $\pm$ 2 ns.

The calculated time difference results from the proper time integrals for the flights to and from Thule, plus the small

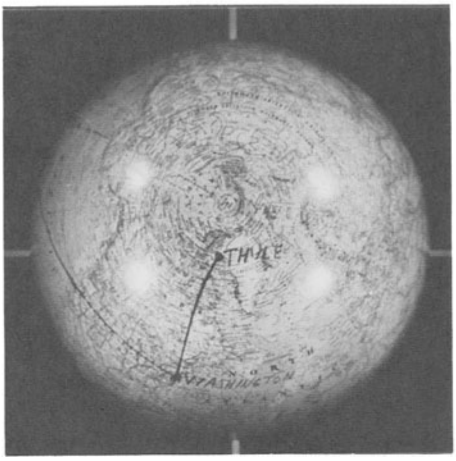


Fig. 57. Aircraft route from Washington, D.C. to Thule, Greenland.

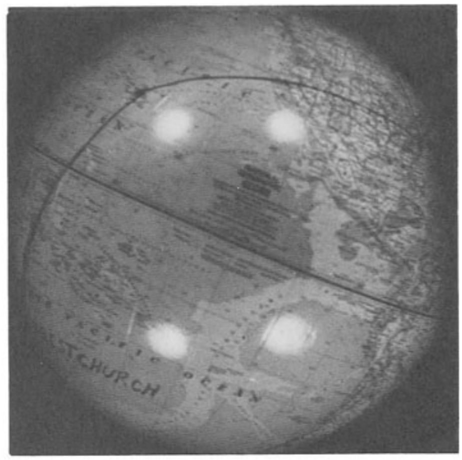


Fig. 58. Path of aircraft from Washington, D.C. to Christchurch New Zealand.

contribution from the different heights above the geoid for Washington and Thule. If the gravitational potential effect is ignored, a rate of 56 ns/day between the two locations is predicted which would yield a difference of 224 ns for the four day dwell time at Thule. This is clearly ruled out by the data. The experiment can be regarded as another measurement of the effect of gravitational potential on proper time, in complete support of General Relativity.

The change in distance from the earth's spin axis between Washington and Thule, which is proportional to the change in surface velocity, is shown clearly on the photograph of a globe from above the north pole in Figure 57.

GLOBAL TRANSPORT OF CLOCKS BETWEEN NORTHERN AND SOUTHERN HEMISPHERES

The Earth Itself as Einstein's Freely Falling Laboratory

We have become accustomed to the idea that there is a local cancellation of the gravitational acceleration of the earth in a space vehicle falling freely to the earth while orbiting around it, as we discussed earlier in developing the ideas of the Principle of Equivalence. A similar local cancellation of the gravitational acceleration of the sun should occur for the earth, falling freely in its yearly solar orbit. Since the gravitational acceleration is given by the negative gradient of the gravitational potential, the local cancellation amounts to subtracting the linear term in the expansion of the potential about the center of mass of the earth (strictly the earth-moon system, but we shall ignore this complication). When this is done, there remain the second order terms in the sun's potential, giving the tidal effects on the oceans and also the solid earth. There remains also, of course, the gravitational potential of the earth itself, whose influence on clocks we have measured. There is a very clear analysis of this situation by J. B. Thomas [22].

What about the effect of the sun's gravitational potential on clocks on the earth? From the historical point of view which we have taken in this paper in terms of the Principle of Equivalence, the answer seems obvious: there should be only second order, or tidal effects, which are very small. Between the ends of an earth diameter pointing to the sun, $\Delta\phi_{sun}/c^2 \approx 7 \times 10^{-18}$ or 0.62 picoseconds per day in its effect on proper time differences, much too small to measure with present day clocks.

However there has been some confusion in the literature on the subject. If the linear term in the sun's potential is thought to be effective on the earth, there should be a day to night shift in clock rates of about 7.8×10^{-13} or 67 ns/day at the equator at the time of an equinox. This question was studied carefully by Professor Banesh Hoffmann in 1957[23] who concluded that no such effects would be measured in the reference frame of the earth. They would be observed in the reference frame of the sun. He called for experiments to check the prediction when clocks of sufficient accuracy became available.*

In 1976 the question was examined again by Professor Roman Sexl[24] who was seeking an explanation for an erroneous report of a dependence of atomic clock rates on latitude.[25] He retained the linear term and concluded there should be a seasonal effect caused by the 23.5 degree inclination of the earth's spin axis to the plane of its orbit. If τ_1 and τ_2 are proper times at latitudes β_1 and β_2, respectively (southern latitudes taken as negative), his proposed relation is

$$\frac{d\tau_1}{dt} - \frac{d\tau_2}{dt} = 14.8[\sin\beta_2 - \sin\beta_1] \cos \left[\frac{t - 21\text{ June}}{365}\right] \text{ ns/day} \quad (58)$$

* The title of Professor Hoffmann's paper was "Noon-Midnight Red Shift". He has told me privately that he was questioned by government authorities at the time of publication on the nature of this periodic shift of political allegiance!

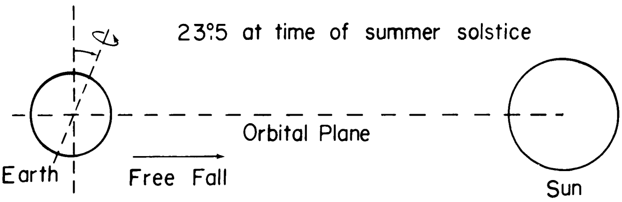


Fig. 59. The earth as Einstein's freely falling laboratory.

The 23.5 degree tilt of the spin axis means that at the time of the summer solstice, the northern hemisphere can be regarded as the "floor" and the southern hemisphere as the "ceiling" of Einstein's falling laboratory as shown in Figure 59. The positions would be reversed at the time of the winter solstice. Clock transport between hemispheres at these times amounts to moving a clock between floor and ceiling of the freely falling laboratory.

Experimental Tests Between Washington, D.C. and Christchurch, New Zealand

Figure 58 of a globe tilted at 23.5 degrees shows the path taken by the clock-carrying C141 aircraft between Washington, D.C (latitude 38° 48') and Christchurch, New Zealand (latitude -43° 29'). The picture also gives an idea of the difference in average distances of these locations from the sun. Two round trips were made in 1977: from 10 July to 17 July, and from 23 July to 30 July. Because of some experimental difficulties in preparing for the flights, they slipped from the planned schedule of just before and just after 21 June. Three days were required for the round trip, including 12 hour stopovers at Hickam Field, Hawaii each way, and a 2 hour stopover at the Travis Air Force Base in California on the trips to the west. On each trip there was a dwell time of four days at the "Operation Deep Freeze" Antarctic support base in Christchurch. (Heating equipment for the plane was needed in New Zealand and air conditioning was required in Hawaii!)

The results of the clock flights are shown in Table 1. There is no evidence for the effect of an alleged linear term in the expansion of the solar gravitational potential. Professor Sexl agrees with this result and has acknowledged (by private communication) that his paper was not correct for observations carried out in the reference frame of the earth.

Table 1. Flights Between Washington and Christ Church

	FLIGHT 1 (ns)	FLIGHT 2 (ns)
$(\tau_A - \tau_G)$measured	115 ± 10	131 ± 10
$(\tau_A - \tau_G)$calculated	129 ± 2	122 ± 2
(Measured - Calculated)	-14 ± 12	11 ± 12
Calculated Effect of Linear Term	80 ± 2	70 ± 2

A breakdown of the calculated proper time integral for the various segments of the flights emphasizes the effect of the cross product term due to the earth rotation given in equation (53). This is displayed in Table 2.

Table 2. Effect of Earth Rotation

	FLIGHT 1 (ns)	FLIGHT 2 (ns)
Andrews AFB to Travis AFB (E → W)	35	31
Travis AFB to Hickam AFB (E → W)	35	31
Hickam AFB to Christchurch(E → W)	47	52
Christchurch to Hickam AFB (W → E)	16	15
Hickam AFB to Andrews AFB (W → E)	-1	-4
Dwell Time on Ground	-3	-3

The aircraft speed was typically 500 knots at an altitude of 35000 feet.

ENGINEERING APPLICATIONS OF ATOMIC CLOCKS REQUIRE AN UNDERSTANDING OF TIME IN GENERAL RELATIVITY

A very significant "milestone" in the development of physics and technology has now been reached, in the opinion of the present writer. The remarkable stability and reliability of contemporary atomic clocks has led to their use in many navigation and timekeeping systems. It is essential for the correct performance of some of these systems that the influence of gravitational potential and motion on proper time be thoroughly understood and included in their design and operation.

The most significant such system, which also produces the largest General Relativistic effects on time, is the Global Positioning System (GPS) which will be used by the U. S. Department of Defense. It will have 18 artificial earth satellites (called the NAVSTAR satellites) in 12 hour period

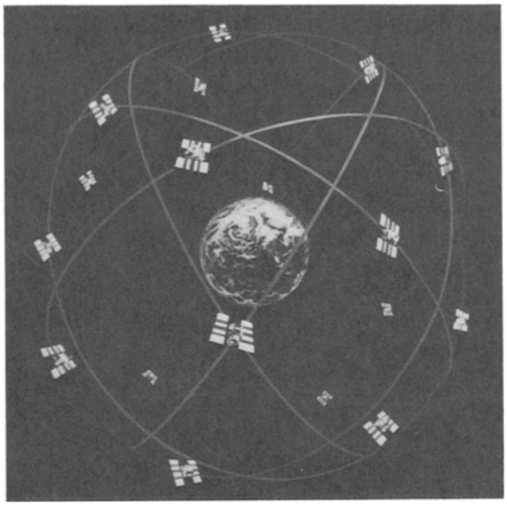


Fig. 60. Artist's conception of the NAVSTAR/GPS.

circular orbits, each carrying very stable atomic clocks and L band microwave transmitters. Figure 60 is an artist's conception of the NAVSTAR/GPS system showing three satellites in each of six equally spaced orbital planes . Figure 61 indicates how the system works. A specially coded bit stream is transmitted whose rate is set by the onboard clock at 10.23MHz. Information about the satellite's orbit is included in the data transmitted. The user, which could be a ship, plane, or even a foot soldier, carries equipment to receive signals from four or five satellites at the same time or in rapid sequence. The receiver electronics includes the same code as is being transmitted. By moving the code forward or backward in time to place it in synchronization with the transmitted signal, the time delay from the satellite is determined. Microprocessors in the receiver can then compute the location of the user in three dimensions with an uncertainty of less than 10 meters, as well as read out the "GPS Time".

The height above the earth's surface of the satellites is 14,000 kilometers. The effects on the clocks can be calculated to first order from equation (36),

$$\frac{d\tau_{Sat}}{d\tau_{Gnd}} = 1 + \frac{\phi_{Sat} - \phi_{Gnd}}{c^2} - \frac{v_{Sat}^2 - v_{Gnd}^2}{2c^2} \tag{59}$$

$$= 5.1 \times 10^{-10} = 44{,}000 \text{ ns/day.}$$

In order for there to be a common GPS coordinate time, this large relativistic effect must be allowed for. Each satellite clock

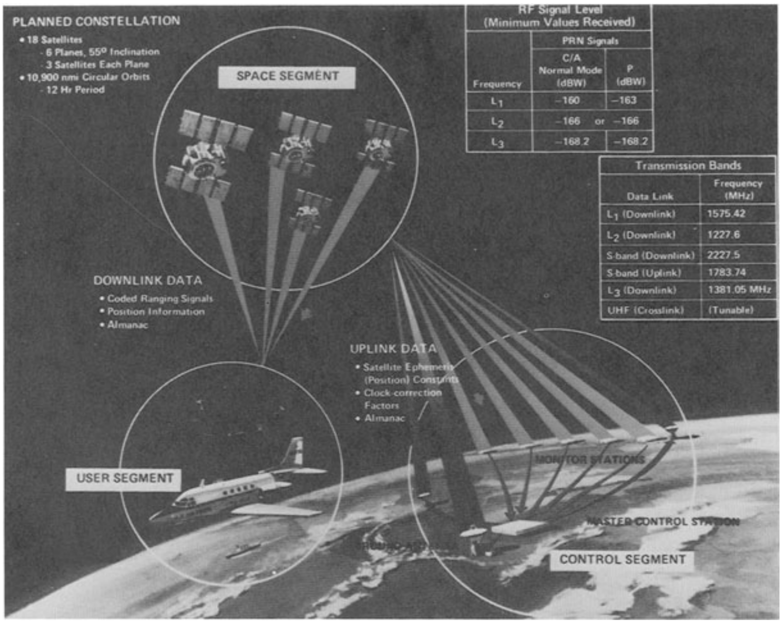


Fig. 61. How the Global Positioning System works.

must be adjusted so that when it is next to the GPS master clock (located at the earth's surface), it will run slow by 44,000 ns/day. Then, when it is placed in orbit, it will run in synchronization with the GPS master clock.

There was considerable uncertainty among the Air Force and contractor personnel designing and building the system whether these effects were being correctly handled, and even, on the part of some, whether the effects were real. This last group was not satisfied until a gravitational frequency shift was measured with a GPS test satellite called NTS-2 by a group at the Naval Research Laboratory in 1977.[26]

If there is some eccentricity to the orbit, there will be a periodic change in the distance of the satellite from the center of the earth. For an eccentricity of 5×10^{-3}, the relativistic effects produce an amplitude of 12 ns in the oscillation of proper time about its average value, giving a peak-to-peak variation of 24 ns. This would produce an excursion of 24 feet in position location on the earth, since in the one-way transmission mode of operation from the satellite, one cannot distinguish a change in time from a change in range. The time-keeping electronics on the satellite is programmed to compensate for these effects of eccentricity.

A common mistake in dealing with relativistic time was also made by one of the Air Force contractors in relation to the GPS. This is the notion that electromagnetic radiation changes frequency (or a photon changes energy) as it propagates through a gravitational potential difference. If the physical clock adjustments have been made as described above so that all clocks are keeping a common coordinate time, then there is no effect on the frequency of radiation as measured in that coordinate time. However, the contractor had included in the computer programs to operate the system just such a correction, effectively correcting twice for the relativistic effects. Actual experience with test GPS equipment is orbit was required to persuade some engineers and physicists of their error.*

We should not be surprised at such lack of understanding of some of the fundamental concepts of General Relativity since the subject is almost never taught to engineers and rarely even to physicists. Also, confusion about these concepts is not restricted to engineers and others who must deal with ultra-stable clocks, but is widespread even among eminent physicists. Consider the following excerpts from Relativity Re-examined by Léon Brillouin (1970)[27]:

> "...All the clocks at rest in our inertial frame will give the same frequency definition with or without gravity potential. The gravity red shift is only due to the motion of photons."

Our experiments clearly contradict this statement. To his credit, at another place in the book, he wrote:

> "...[improved atomic clocks] would allow us to perform many important experiments that would tell us definitely what to think of relativity!"

If Professor Brillouin were still living, perhaps he would accept our experiments as convincing evidence for the correctness of Einstein's views on time.

* It had been pointed out and explained to them by Leonard Cutler, Gernot Winkler and the author during a presentation at the GPS Program Office of the results of the local aircraft experiments described in this paper.

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   The author's chief collaborators in the planning and execution of the experiments were Dr. Leonard Cutler of the Hewlett-Packard Company, and Dr. Gernot Winkler of the U. S. Naval Observatory. Two other physicists who contributed much to the experiments are former University of Maryland, graduate research students, Dr. Robert Reisse, now at the University of Arizona and Dr. Ralph Williams, now at the Texas Instruments Company. The work of another University of Maryland graduate student, Dr. John Degnan, now at the Goddard Space Flight Center, on the short pulse laser used in the measurements was very important.

   The local experiments were performed during the period May 1975 through January, 1976 at the Patuxent Naval Air Test Center in Maryland with the support of the U. S. Navy. The global experiments were performed during the period May through July, 1977 from the Andrews Air Force Base in the Maryland suburbs of Washington, D.C. and were jointly supported by the U.S. Air Force and Navy.

   Space does not permit the acknowledgment of the esential contributions made by the engineers and technicians at the University of Maryland, the Hewlett-Packard Co., and the U. S. Naval Observatory. These are listed in the reference cited above.
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